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Option :

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Par :

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THESIS

A dissertation submitted for the degree of **DOCTORATE (LMD)**

Study of the cracking of a structure subjected to thermal loading using the mechanistic-reliability approach

Option :

Mechanical Reliability

By :

Mustapha HADJI

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Dedication

I dedicated this modest work to my parents, my wife, and my whole family. Your love is the fuel that kept me going, and your faith in me made this work possible.

Abstract

In hot aluminum extrusion, die lifetime is a critical industrial issue governed by thermal fatigue and damage mechanisms. This study proposes a probabilistic mechanistic–reliability framework to predict extrusion die lifetime using stress–strength interference analysis. The approach couples the ***Hansel–Spittel*** rheological model with the ***Lemaitre–Chaboche*** fatigue damage model to account for material behavior and damage evolution. Reliability analysis is used to evaluate the influence of random process parameter variations on die performance. The results highlight the dominant effect of temperature on damage accumulation, ***reliability index β***, and the maximum number of extruded billets, denoted as **Nf**

.**Keywords:** Reliability, Aluminum extrusion, Critical Damage, Die Lifetime.

ملخص

يُعد عمر قالب البثق في صناعة بثق الألومنيوم الساخن قضية صناعية رئيسية تتحكم فيها آليات التعب الحراري والتلف. تقدم هذه الدراسة إطارًا احتماليًا ميكانيكيًا-موثوقيًا للتنبؤ بعمر القالب اعتمادًا على تحليل تداخل قوة الإجهاد. يعتمد النهج على اقتران النموذج الريولوجي لـ Hansel–Spittel مع نموذج تلف التعب لـ Lemaitre–Chaboche لتمثيل سلوك المادة وتطور التلف. يسمح تحليل الموثوقية بتقييم تأثير تقلبات معلمات العملية العشوائية على أداء القالب. وتُظهر النتائج الدور الحاسم لدرجة الحرارة في تراكم التلف، ومؤشر الموثوقيةβ ، والعدد الأقصى للبلتات المبثوقةN_f

.

الكلمات المفتاحية: الموثوقية، بثق الألمنيوم، الأضرار الجسيمة، عمر القالب

Résumé

Dans l’extrusion à chaud de l’aluminium, la durée de vie des filières constitue un enjeu industriel majeur, principalement liée aux mécanismes de fatigue thermique et d’endommagement. Cette étude propose une approche probabiliste mécanistique-fiabilité pour la prédiction de la durée de vie des filières basée sur l’analyse d’interférence contrainte-résistance. Le modèle rhéologique de ***Hansel–Spittel*** est couplé au modèle d’endommagement en fatigue de ***Lemaitre–Chaboche*** afin de décrire le comportement du matériau et l’évolution des dommages. L’analyse de fiabilité permet d’évaluer l’influence des variations aléatoires des paramètres du procédé. Les résultats mettent en évidence l’effet dominant de la température sur l’endommagement, ***l’indice de fiabilité β*** et le nombre maximal de billettes extrudées **Nf**.

Mots clés : fiabilité, extrusion d'aluminium, dommages critiques, durée de vie de la filière.

Summary

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List of Symbols and Abbreviations

R(t)	Reliability function
t	Time
Ui	Gaussian variables
T	Rosenblatt transformation
Xi	Basic variable
H (Ui)	State function in reduced normed space
G	Limit state function
T-1	Rosenblatt inverse transformation
B	Hasofer-Lind reliability index
σ	Equivalent stress
A	Coherence
ε	Equivalent strain
ε•	Strain rate
T	Temperature given in Celsius
m1 and m9	Define the sensitivity of the material to temperature
m5	Coupled term temperature and strain
m8	Coupled term temperature and strain rate
m2, m4 and m7	Define the sensitivity of the material to strain
m3	Depends on the sensitivity of the material to strain rate
N f	Structural life
σu	Ultimate strength of the material
Jmax	Maximum value of the VonMises invariant of the stress tensor
α and β	Material coefficients
ΔJ	Stress deviator
$1\ I$	Mean value of the first invariant of the stress tensor
$_{1}(\ )\ _{l}\sigma\ I$	Mean cyclic stress range
$\big(\ \big)\ _{1}M\ I$	Mean stress effect
b, σl0	Material coefficients
D	Damage evolution
n	Number of applied cycles
Dc	Critical damage
P*f*	Probability of failure

GLOBAL
INTRODUCTION

Global Introduction

In a world characterized by rapid changes and significant technical advancements, the global aluminum extrusion market has seen considerable growth. In the realm of extruded aluminum manufacturing, related to our research paper, extrusion tools and molds have garnered significant attention due to their role in enhancing efficiency and reducing costs, encompassing material expenses, research expenditures, and manufacturing costs with thermal processing. up to 10% of the profile production cost is attributed to tooling. Therefore, the lifetime of the die is always a concern and is often poorly estimated given various production factors which remain insufficiently controlled, despite the efforts that have been made over the last years in the development of cutting-edge technologies to make the manufacturing process precise, reliable and efficient.

During the extrusion process, the dies endure variations in temperature due to intermittent contact with the hot metal. The variations in temperature cause thermal stresses in the die material, which can result in the formation and spread of cracks due to thermal fatigue. This damage phenomena is a significant form of deterioration that impacts the life cycle of extrusion dies. It can result in large maintenance expenses, production interruptions, and, in severe instances, catastrophic failures.

. The lifetime of extrusion shaping dies is defined by the number of extruded billets in accordance with the requirements of the technical drawing. However, it remains closely dependent on the geometric shape and dimensions of certain extruded profiles in the die. Our research is dedicated to enhancing the quality of metals and controlling their properties to extend die life.

Die life is primarily determined by the number of billets extruded in relation to profile size requirements, making it a critical concern. Controlling die failure mechanisms, such as thermal fatigue and surface wear, is essential for achieving economically viable tool longevity. Additional load effects may also be caused by fatigue damage [1]. 70% of die-casting die failures are caused by thermal fatigue, one of the several failure modes. [2], stands out.

The user may choose material parameters, especially crack growth statistics, from a menu, which is a handy feature of the program. The kinetics of crack formation can be represented by a variety of models. [1]. Compared to other steels in its class, AISI H13 is a hot work tool steel containing 5% chromium and a higher than average amount of vanadium By virtue of its high hardenability, material chemical composition presented in Table.1 [3.4]. It exhibits a combination of desirable properties including toughness, strength, ductility, and thermal conductivity [5,6]. Consequently, H13 tool steel finds extensive applications in casting, extrusion, and forging processes. Its resistance to thermal fatigue cracking, a result of repeated heating and cooling cycles in hot working setups, is attributed to its hot hardness, which is synonymous with hot strength [7].

The value of the first effective strain, which is a critical parameter that requires careful evaluation, is closely related to the effect of temperature on die life [8]. The most pronounced die wear occurs near the die bearing, posing a higher risk of die failure due to significant plastic deformation, elevated die temperature, and interface pressure [9]. During the hot extrusion cycle, especially under high-temperature and high-speed conditions, localized heating due to friction at the billet-tool interface results in a temperature increase close to the melting point, accompanied by high tension stress [10]. Improving and controlling scrap production and assessing its impact on product quality is dependent on variables related to extrusion temperature, time, ram speed, pressure, and die geometry [11].

Aluminum alloy 6063 with main chemical composition of 0.47% Si and 0.55% Mg is known to be one of the properties that influences the die's life during the extrusion process. This is due to the metal's quality, particularly when utilizing the secondary melting alloy especially from mixed scrap, which enhances the quality of the coefficient of friction's reduction. This raises the temperature, which in turn shortens the die's lifetime.

Several vital parameters, including equipment conditions, operating conditions, temperatures, pressures, and die quality, significantly influence the extrusion process. Despite diligent efforts by manufacturers, various challenges in the process still lead to product defects [12]. Many defects and wastage are linked to the choice of billet size for extrusion and involve factors such as surface quality, temperature, speed, die geometry, and weld joints. To mitigate these issues, it is imperative to maintain control over all relevant parameters, including extrusion billet size, cutter position, puller speed, die entry angle, conveyor roller surfaces, temperature, and speed [11].

In light of the aforementioned considerations and recent research on extrusion die behavior and defects, the primary objective of our research paper is to determine the reliability index by considering die material elements and their influencing factors. To achieve this goal, we have developed a model to analyze the behavior of the extrusion process by integrating mechanical reliability and key influencing factors. This has led to the development of a mechanical model that combines the rheological model with the damage model.

In this context, the thesis is done. The work of this thesis has also been divided into four chapters:

Chapter one provides a critical review of the state-of-the-art in the reliability of thermal fatigue damaged structures, with a particular focus on aluminum extrusions. It explores the basic principles of thermal fatigue, experimental and numerical methods to characterize and model this phenomenon, as well as lifetime assessment methodologies. In addition, current challenges and limitations are highlighted, highlighting the need to develop more robust and accurate methods to predict the reliability of extrusion dies in the presence of thermal fatigue damage.

In the second chapter, the reliability approach to the structure allows us to understand the risks taken. In the analysis of the reliability of structures, the influential parameters are considered as probabilized variables and the probability of failure and inspections, the lifetime, etc. are calculated. Feedback and knowledge of the kinetics of degradation are two of the essential conditions for applying structural reliability analysis.

Structures age, the properties of materials may be altered, operating methods are no longer the same.

The probabilistic approach to reliability of structures then proves essential, the risk is evaluated in the form of a probability and no longer in the form of a binary judgment (the design is acceptable or not, the operation can be continued or not).

Calculating this probability makes it possible to reduce the risk of failure by organizing maintenance and inspection programs and to extend the operating life by optimizing their use.

Chapter Three presents the important tools of linear mechanics of fracture and fatigue, which are widely used to describe fatigue cracking phenomena. This part will help us build an approach to be followed in the rest of our study.

Mechanisms deteriorate severely during operation, which can lead to mechanical and thermal damage, ultimately compromising their performance and service life. Mechanical damage can appear in various forms, including abrasive wear, corrosion, and plastic deformation. high temperature and rate dependent material behavior for aluminum extrusion with reliability prediction.

These mechanisms are mainly influenced by the pressure applied, extrusion speeds and the friction at the die entrance The prediction of die life's accuracy and die's dependability becomes a fundamental concern in the maintenance schedule so that downtimes are minimized and product uniformity is guaranteed. Conventional age estimation approaches are often deterministic, thus neglecting the probabilities involved in the loading conditions, properties of materials and the manufacturing processes. To overcome this, a mechanical reliability approach is advocated which utilizes probabilistic concepts and complex stress-strain modeling. Incorporating Hansel-Spittle rheology, Lemaitre-Chabuch degradation, and probabilistic reliability techniques, the authors developed a common methodology to assess random process variations on die performance.

The die life estimation part of the predicted maintenance model will integrate advanced Hansel and Spittle fabrication technology models and a probabilistic die lifing analysis. This method will enable a better and practical prediction of die functioning and its remaining life as it will consider anthropogenic variations of materials, processes and even geometrical limits of tolerances that tend to cause variation of die characteristics in course of operation. By propagating such uncertainty, this enhanced nursing framework will deal with risk management and the embedding adaptive tumor shaping in the planning processes of dies reconditioning and replacement.

Finally, appropriate model will be built to consider behavior of the die, the most critical part of the tools in the process of aluminum extrusion, and combine it with mechanical reliability in order to find the different key factors. This then leads to the initiation of a mechanical model that incorporates the rheological model with the aluminum extrusion process.

CHAPTER I
LITERATURE REVIEW

Chapter I

I.1. Introduction

The automotive and aerospace industries, due to the lightweight and high strength properties of aluminum, have been increasing its use in their applications [13]. It is into this trend that the extrusion process in the production of profiles for many applications with these materials makes use of their mechanical properties and lightness. At the European level, this sector involves more than 40,889 employees and nearly 2,700 companies, with more than 100,000 jobs and a total production of more than 2.919 million tons of aluminum at the international scale. The present-day market for the extrusion sector demands presses that should have the capability to produce a large variety of sections at varying speeds and programs. For instance, when the manufacturer has to develop a new profile, it has to test the same in order to meet the mechanical and dimensional properties. However, the reliability of these aluminum extrusion dies under thermal fatigue is an aspect of major concern. Several factors may contribute to causing the failure of aluminum extrusion dies that are subjected to fatigue due to thermal cycles. Some of the factors include: - Temperature variations during extrusion - Changes in stress levels due to thermal expansion and contraction - Microstructural changes and phase transformations - Accumulation of thermal strains - Formation of cracks and propagation of defects - Inadequate cooling mechanisms. To identify these issues and enhance the reliability of aluminum extrusion dies, there have been quite a few research studies. [14,15]

I.2. State of the art for the aluminum extrusion dies lifetime

I.2.1. Die Design and Manufacturing Processes

Optimal die design includes die profile, land length, bearing length, and entry angle. These parameters affect metal flow, temperature distribution, and extrusion speed associated with the ultimate product quality. [16,17]

Research by various authors has been directed toward optimum design to improve die reliability. For example, Geiger and Haensel used the FEM simulation in optimization studies related to extrusion die tool loading and fatigue crack initiation, where methods were presented for the optimization of the tool and improvement of life [18].

Another study by Iwama and Morimoto [17] reported that heat cracks and the softening of the surface layer had reduced the die life of a backward extrusion punch during conventional processing conditions and suggested a dry lubrication adhesion layer to improve die performance.

Other studies have concentrated on using deformable dies along with simulation for enhancing die reliability. For instance, Assaad used finite element methods in simulating the aluminum extrusion process with a deformable die, and he indicated that 3D simulations are very important for such complex processes [19]. Geiger and Haensel [20] applied FEM simulation of loading and fatigue crack initiation in extrusion dies to suggest methods of tool optimization and improvement of tool life, including strategies to mitigate thermal fatigue failures. This indicates how important advanced simulation techniques are in optimizing die design and improving the life of tools under conditions of thermal fatigue.

I.2.2. Die Material Selection

Paper by Tekkaya and Sonsoz (1995) [21]. The authors studied the fatigue behavior for extrusion dies. They firstly calculated the effective stress intensity factor at different locations of the die inlet using the finite element method then applied the Paris/Erdogan fatigue law in order to simulate the crack growth and estimated the life of the extrusion die. The authors managed to simulate experimentally observed behavior of the crack growth, including the stable-unstable-stable growth with final fracture. [20].

Selection of the proper die materials is only one aspect of ensuring that die performance provides for minimal defects. H13 and H11 hot-work tool steels remain the basic materials for aluminum extrusion dies because they have high strength, toughness, and resistance to wear. Advanced die materials, such as nickel-based alloys and powder metallurgy tool steels, are used under more stringent conditions of extrusion or where the nature of service demands. Research, at present, is in the direction of finding new die materials with improved mechanical and tribological properties for better performance of dies.

Common die materials are H13, H11, and tool steels because of their high hot hardness, wear resistance, and thermal conductivity. [15,16]

I.2.3. Aluminum Extrusion Die Failure and Defects Modes

The several failure modes and defects of an aluminum extrusion die has enveloped during installation and operation of it. In this paper, a general understanding of all sorts of failure modes and defects that can occur to an aluminum extrusion die during its operational and installation processes with some effective corrective and preventive measures will be looked towards. - (Matienzo et al., 1983) [22].The researchers conducted experimental tests and analysis to determine the impact of factors such as radial clearance percentage, punch and die profile radii on the likelihood of failure and the occurrence of defects (Zaid, 2016) [23].

I.2.3.1. Failure Modes in Aluminum Extrusion Die

The failure modes in aluminum extrusion die can be divided into different categories based on the nature of the failure. These are mechanical failure, thermal failure, and wear-related failure.

Principal causes could be stress concentration, material fatigue, and insufficient strength to support extrusion. Due to too much heat during the extrusion process, failure caused by thermal effects could happen, resulting in thermal deformation and cracking. Wear failure could occur as a result of abrasive wear, adhesive wear, and corrosion..

I.2.3.1.1. Mechanical Failure

Mechanical failure of an aluminum extrusion die may be due to fatigue, overload, or even some types of material defects. When this kind of failure occurs, the cracking or fracturing can happen on the external part of the die and can even result in complete breakage. The normal sign of fatigue failure is that the die goes through repeated cycles of loading and unloading that progressively damage the

This results in compromising the structural integrity of the material, leading to failure. Overload failure could happen when the force applied on the die is greater than its designed capacity—it may bend or totally fail.

Material imperfections, such as inclusions and voids, may become an added source of mechanical failure since their presence creates loci for the concentration of stress capable of breaking the structural integrity of the die.

I.2.3.1.2. Thermal Failure

Thermal failure is typically caused by excessive heat or thermal cycling, leading to thermal fatigue, softening of the die material, and ultimately, deformation or failure of the die.

During the extrusion process, the intense heat generated can cause thermal expansion and contraction, leading to thermal stresses.(Yadav et al., 2020) [26] These thermal stresses can promote crack initiation and propagation, especially in areas of the die that experience high temperature differentials (Erenkov et al., 2020) [25].

In a research work presented by Tekkaya and Sonsoz in 1995 [21], the authors had investigated the fatigue behavior of cold extrusion dies. They had calculated the stress intensity factor at the points of the die inlet by using finite element analysis techniques. Thereafter, the authors had utilized the Paris/Erdogan fatigue model to compute the crack propagation life of the extrusion die and predict its life. The study was able to successfully simulate the experimentally observed crack growth behavior. It shows a sequence of unstable growth until fracture. This work testifies to the fact that for an estimation of the life of extrusion dies under thermal fatigue conditions, one needs to resort to advanced methods of modeling.

I.2.3.1.2.1. Fatigue Cracking:

Li et al. (2013) [27] investigated the effect of process parameters on die wear behavior, including thermal fatigue, in the extrusion of aluminum alloy rod. FEM simulation was used in this work to analyze the mechanisms for die wear and provide suggestions based on their findings toward improved design optimization for dies. The paper discussed how to improve the die design as well as the reliability by understanding the influence of process parameters on die wear and thermal fatigue.

I.2.3.1.2.2. Fatigue Crack Initiation propagation Life:

Defining the thermal fatigue crack initial (TFCI) existence thru the probabilistic behavior of tooling service existence in commercial aluminum extrusion, a observe through Sheikh et al. (2004) [16] investigated numerous die screw ups related to numerous profiles. The aim become to establish the relationship among die reliability and the complexity of the profile.

Zhong et al. (2013) [28] developed advanced finite element models to simulate thermal fatigue crack propagation in extrusion dies. Their research focuses on an understanding of the mechanisms responsible for crack propagation and failure. The aim is to study these processes in detail to derive strategies that prevent thermal cracking and improve the durability of dies.

Iwama and Morimoto showed that the die life of a backward extrusion punch is shortened under the conventional processing condition owing to heat cracks and softening of the surface layer. Therefore, in their process, they used a dry lubrication adhesion layer which binds strongly to the die to lessen the problem of thermal fatigue. Advanced surface treatments no doubt would definitely play an important role in improving die performance and also reducing thermal fatigue as stated.

I.2.3.1.3. Wear-Related Failure

Abrasive wear, adhesive wear, or corrosive wear in a die may cause wear-related failure. Each of these types of wear necessarily causes deterioration of the surface and loss of dimensional accuracy.

Abrasive wear results from the contact and disintegration of the die surface by hard particles or debris during the course of the extrusion process, while adhesive wear involves the transfer of material between two contacting surfaces resulting from a relative motion on a rubbing surface, leading to degradation of the surface. Corrosive wear means that a die is exposed to some kind of corrosive substance, namely chemistry or an aggressive environment. Both cause degradation and loss of surface material.

I.2.3.2. Defect Modes in Aluminum Extrusion Die

Akhtar and Arif [24]. presented the fatigue failure of an extrusion die. The process parameters and design features affecting the life of the die were analyzed, emphasizing that understanding the mechanisms of fatigue failure is important for improving die design and its reliability.

Thermal fatigue, mechanical fatigue, abrasive wear, and chemical wear are the other causes of die failure. Understanding these mechanisms of failure would ensure the efficiency of preventive measures in improving die life. [16,17,31]

The most common failure modes of extrusion dies include thermal fatigue, mechanical fatigue, abrasive wear, and chemical wear [16,24]. A proper understanding of all these modes is crucial for extending die life and preventing defects.

However, due to the fact that almost 100% of dies fail because of fatigue, fatigue failure is a major concern. Several experimental investigations have been done which demonstrate that the crack growth in extrusion dies has an interesting behavior; namely, the stable-unstable-stableunstable phases. Such a crack growth behavior has been analyzed based on analytical and numerical methods using linear elastic fracture mechanics[24].

Finite element simulations have been used to model the stress state at the crack tip and its evolution with increasing crack length. The results indicate that the stresses around the crack tip drop steeply with increasing crack length, while the stress intensity factor (especially KII) increases due to the reduction in cross-section. This explains the observed crack growth behavior.

Computational studies have also shown that the location of crack initiation can significantly affect the die life, with variations from 7,000 to 18,000 workpieces produced. This helps explain the large scatter in experimental fatigue life data observed by researchers[24].

Defects in aluminum extrusion die can be classified into various modes, including surface defects, internal defects, and dimensional defects.

I.2.3.2.1. Surface Defects

Surface defects: These refer to irregularities such as scratches, dents, or other flaws that compromise the surface smoothness of extruded aluminium profiles. Defects may result from inadequate die design, insufficient die surface treatment, or wear-induced failure. Internal Defects Internal defects refer to the imperfection or irregularity within an aluminum extrusion die. These include inclusions, voids, cracks, or improper grain structure. These defects can arise from various sources such as poor material quality, improper die design or maintenance, or incorrect extrusion process parameters (Zaid, 2016) [23].

I.2.3.2.2. Internal Defects

These internal defects can appear as porosity, inclusions, or internal cracks within the extruded profile, and reduce the mechanical properties of the extruded aluminum material. Defects can be created with consideration of several resources of origin such as inappropriate die design, unsuitable treatment of the die surface, and unsuitable parameters of the extrusion process, Erenkov et al., 2020 [25]. Besides, dimensional defects may appear, such as incorrect shape or dimensions of the profiles being extruded from the aluminum extrusion die.

I.2.3.2.3. Dimensional Defects

Dimensional defects involve those geometrical or dimensional inaccuracies in the extruded profiles due to die wear, improper design of the die, and instabilities in the process. Such faults might lead to product failure or customer discontent about the overall quality and performance of extruded aluminium profiles. Defects in aluminium extrusion dies can result in various failure modes, hence undermining the quality and performance of the extruded profiles.

I.2.4. Die defect analysis and Life prediction

The defects related to aluminium extrusion dies, like surface cracks, die deflection, and premature wear, seriously reduce the quality and productivity of products. For the prevention and defect control, efficacious design, appropriate material selection, and proper manufacturing techniques are required.

Surface cracks, die deflection, and premature wear are just a few of the most common defects in aluminum extrusion dies that are dealt with in order to understand the underlying mechanisms of die failure. The authors review the causes related to thermal stresses, cyclic loading, and material incompatibility and their interaction with die wear processes. The study has emphasized the proper design of dies, selection of material, and manufacturing processes in order to avoid or control these defects and prolong die life. Defect prevention methods are discussed in relation to optimization of die geometry, appropriate selection of die materials which have high strength and wear resistance, and modern manufacturing techniques and surface treatments.

I.2.5. Probabilistic Study

I.2.5.1. Evolution of reliability concepts :

The introduction of reliability analysis in the field of engineering first occurred in 1947 when Freudenthal presented the fundamental problems of structural safety for a specific construction detail under variable loading. In 1949, Lévi [32] published the initial probabilistic calculations about the safety of structures. This presentation was subsequently followed by other significant publications, such as Rjanitzyne's in 1949, which first defined a reliability index, later adopted by Cornel [33] and improved by Hasofer and Lind [34]. The publication of Rjanitzyne was followed by those of Johson in 1953 [33], Pugsley in 1966 [35], and Ferry-Borges and Castanheta in 1971 [36]. These individuals have developed the classical theory of reliability and have widely disseminated it. Following this period, there has been a significant number of publications and works related to reliability-based structural analysis. Since 1947, several reliability methods have been developed, such as the FORM and SORM transformation methods, simulation methods like Monte Carlo, and other methods like Markov theory, utilized by Lin and Yang [37] and Lassen and Sorensen [38],...Many works refer to all these methods, such as those by Thoft-Christensen and Baker [39], Madsen et al [40], Melchers [41], and Lemaire [42].

I.2.5.2. Reliability and Productivity:

Improving reliability and productivity in aluminum extrusion is crucial. A study by various authors emphasized the need for efficient die design and optimization to reduce defects and improve overall productivity. For instance, a study by Assaad highlighted the importance of controlling the aluminum flow and optimizing die design to minimize defects and improve product quality [43].

I.2.5.3. Probabilistic Study of Thermal Fatigue Failures:

One such work by Sheikh [16] dealt with the probabilistic behavior of service life of tooling in commercial aluminum extrusion. In this, die failures for different profiles were analyzed to understand how die reliability related to profile complexity. The authors modelled the die life with different distributions of probability and concluded that the normal distribution could therefore be a suitable fit for the data, including thermal fatigue failures. This paper points out how knowledge of the probabilistic nature of die failures is important for improving die design and reliability.

In the industry of aluminum extrusion, thermal fatigue failures are grave concerns because they cause very expensive downtime and safety hazards. [44]. An aluminum extrusion die fails due to thermal fatigue as a result of repeated heating and cooling during production. [45] The thermal stresses induced by these heating and cooling cycles can eventually lead to crack development and material failure. Understanding the nature of thermal fatigue failure and the probability involved will go a long way in devising effective strategies for maintenance and design with regard to improving the lifetime of extrusion dies and reducing loss in production time, as well as ensuring safety. [46] In the present work, a theoretical investigation of the probabilistic nature of thermal fatigue failures is carried out for aluminum extrusion dies with regard to material properties, operating conditions, and geometric features[47]. It is by quantifying the probabilistic behavior of these failures that predictive models and maintenance schedules can be developed to lessen the impact of thermal fatigue and improve the reliability of the aluminum extrusion process. [48].

I.2.6 Reliability analysis of Extrusion processes

Sheykh and al, Terčelj et al [16, 49] have proposed the study to predict wear in hot work tools. The neural network method has been combined with the finite element method in order to take into account random variables related to input parameters and rheological behaviour during an operation of extrusion shaping. The kind of methodology is opposed to conventional empirical statistical methods and hence provides a more effective examination of the process of shaping. However, for neural networks to be adequately precise and efficient when used with the finite element method, the input parameters influencing the process need to be determined with due accuracy. It means taking into consideration the uncertainties of the measurement results due to means and methods used, described by coefficients of variation. These measurement results include chemical composition from spectrometry or diffractometry, mechanical properties from mechanical tests, sampling of temperatures, applied loads, stresses, deformations, and wear measurements.

All the research work presented on the use of probabilistic approaches in the analysis of extrusion and shaping processes agrees that the problem lies in the collection of uncertainties expressed in standard deviations or in correlation coefficients of the values d input, linked to the rheological and mechanical behavior laws, applied stresses and collected deformations. Several artificial intelligence techniques are proposed [49,50,16,19,22] and all contribute to improving the lifetime of the tools and ensure the quality of the formed part. Although they have some very interesting guidelines, they are not that simple to use. Today, there are very powerful design reliability analysis tools, very simple to use and based on the determination of reliability indices using a FORM/SORM approximation approach, presented below, or well by the Monte Carlo approach.

Remember that reliability is the ability of an entity to perform the required functions under given conditions for a given duration. It is characterized by the probability R(t) that the entity E performs these functions, under the given conditions during the time interval [0, t], knowing that the entity is not broken down at time 0[39]. R(t) = P [E not faulty on [0, t]] (I.1)

The first proposal for a reliability index was made by Rjanitzyne in the 1950s, Soviet Union. However, it was Cornell who popularized this idea. Then, various proposals were presented but the most complete form is due to Hasofer and Lind who rely on a rigorous definition, figure I.1 . Hasofer and Lind showed that the measurement of the reliability index had to be taken in a space of standardized Gaussian variables Ui. To do this, we must define a transformation T (**Figure I.1**).

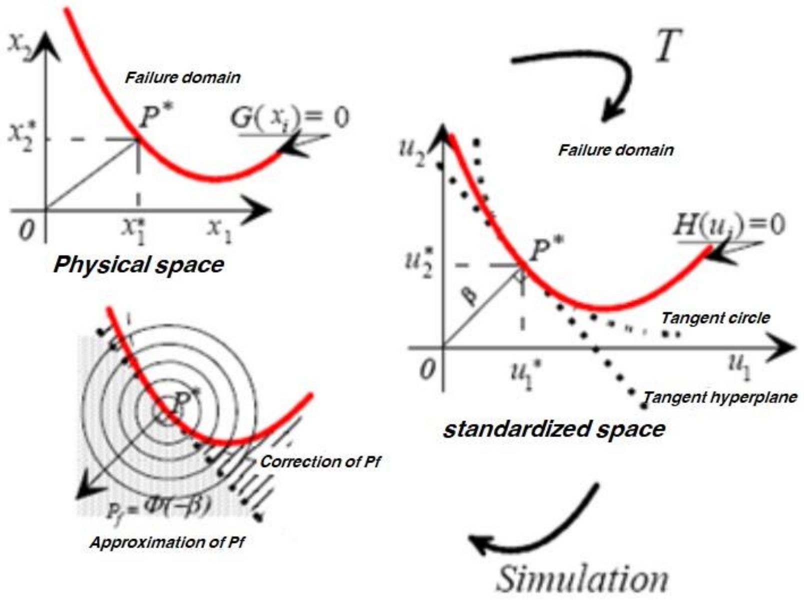


Figure I.1 *: FORM/SORM approximation method, Hasofer-Lind index*

I.2.7. Heat Treatment of Aluminum Extrusion Die

The thermal treatment of aluminum extrusion dies is very important for its life and performance [52]. Proper thermal treatment can significantly increase the hardness, wear resistance, and heat resistance of the die material. Common thermal treatment approach for aluminum extrusion dies includes solution heat treatment followed by aging. During the solution heat treatment, the die is treated at a specified temperature to dissolve different alloying elements or compounds in the aluminum die. This is followed by rapid cooling to "quench" the material. After quenching, the die is then aged at a lower temperature to precipitate the dissolved elements, which increases hardness and strength [53]. Another heat treatment method, which could be applied in the case of aluminum extrusion dies, is precipitation hardening [54]. The process involves heating the die to a certain temperature at which small dispersed precipitates formed within the materials contribute to strengthening. This way, controlling the parameters of this heat treatment allows tailoring the mechanical properties of the aluminum extrusion die so that a specific specification could be met.

Another consideration in the heat treatment process is dimensional stability. During the extrusion process, the die undergoes a very high temperature; thus, thermal expansion and contraction set in. Proper heat treatment can practically eliminate undesirable effects associated with thermal cycling so that after repeated extrusion cycles, the die retains its dimensional integrity [16.18.55.56]. By using different techniques and parameters, the requisite heat treatment of aluminum extrusion dies can be attained.

Apart from heat treatment, further processing, such as surface coating or finishing, can considerably increase the functionality and life of the die. These processes may include treatments such as nitriding, application of hard ceramic coatings, or introduction of wear-resistant materials to the critical areas of the die surface. The choice of an appropriate post-treatment process is made taking into consideration the required surface hardness, wear resistance, and operational conditions of the die [54].

Design and implementation of a heat treatment process plays a role, in extending the lifetime and improving the performance of aluminum extrusion dies to boost efficiency and quality in the extrusion process. It is widely acknowledged that a meticulous approach, to heat treatment significantly enhances the durability and effectiveness of aluminum extrusion dies while also advancing efficiency and product quality [57].

Below is the chart illustrating the heat treatment process, for H13 hot working tool steel that is utilized in creating aluminum extrusion dies. It outlines the phases as follows:

7-1 Preheating:

The die is heated to 1150-1250°F (621-677°C) at a rate that does not exceed 400°F per hour (222°C per hour) and then equalised. It is advised to perform a double preheat for dies that are elaborate [53].

7-2 Austenitizing (High Heat):

The metal mold is quickly heated to a temperature range of 1800 to 1890 °F (982 to 1033 °C). A temperature of 1800 °F (982 °C) provides durability and strength whereas reaching 1890 degrees Fahrenheit (1033 degrees Celsius)) results in increased hardness and better resistance, against thermal fatigue cracking and wear. The mold is maintained at this temperature for a duration ranging from 30 to 90 minutes.

7-3 Quenching:

The die is quenched quickly to below 1000°F (538°C) in oil or pressurized gas. For sections over 5 inches (127 mm) in thickness, accelerated cooling is necessary in order to attain the maximum hardness and toughness[58,59].

7-4 Tempering:

It should be noted that the die is usually tempered immediately after quenching at about 1000-1150°F (538-621°C). The die is held at the tempering temperature for at least 1 h/in. (25.4 mm) of thickness but not less than 2 h. Double tempering is required, and a third temper is often used as a stress relief after finish machining, grinding, and EDM work [58,59].

The specific heat treatment process for H13 aluminum extrusion dies involves vacuum heating to 1030°C, oil quenching, and tempering at 580°C. This process aims to achieve a hardness of 48-50 HRC, providing the die with high hardness, thermal fatigue resistance, and the ability to withstand the demands of modern high-speed extrusion processes[59,60].

In summary, the state of the art for the thermal crack in aluminum extrusion dies lifetime and defects involves probabilistic studies of thermal fatigue failures, thermal fatigue crack initiation life modeling, finite element simulation of thermal fatigue cracks, understanding the effect of process parameters, and strategies for mitigating thermal fatigue through die design optimization and surface treatments.

I.3. Manufacture of alloy profiles 6063 (ALGALPLUS)

I.3.1. Melting workshop

The purpose of the refurbishing equipment is to refurbish the falls, defective products that appear in the process of making the profiles as well as the aluminum waste from the outside to obtain the tickets. The unit has opted for alloys of the 6063 series (Al-Mg-Si) whose properties of its alloys are:

- Very good heat deformation ability that allows the obtaining of complex shapes.
- A sufficient level of mechanical strength to combine with good fit to complex forming.
- A sufficient level of mechanical strength to combine with a good fit for cold forming.
- Good corrosion resistance that can be obtained in the raw press by lacking or anodizing..

I.3.1.1. Melting furnace and casting machine

The unit contains a fusion oven with a loading door on the front side, a heating device on the rear side and visiting doors on the opposite side of the swivel and a casting hole located on the rotating axis of the furnace swivel. The swing is ensured by the ascending and descending movement of a hydraulic valve and it is heated with natural gas. The main characteristics of the furnace are:

- furnace capacity: 500 Kg of Al.
- Melting capacity: 1800 Kg Al/h.
- Casting capacity: 25T/h.
- Oven ambient temperature: 1150°C max.
- Operating condition: 16 H/day..

The refurbishment also contains an aluminum separator designed to recover the melted aluminum contained in the dirt on the background of a U-shaped creuset. In this part you have to control the temperature and agitation as well as the use of a separator agent. The chemical correction is achieved after simultaneous chemical analysis of the alloy elements by a spectrometer.

The liquid metal of the melting and casting furnace passes through the continuous casting machine tube and then through the glass filter. It is introduced by a distribution device in the shells arranged on a shell carriage. Liquid metal solidifies under the effect of cooling water. The main characteristics of the casting machine are :

- The weight of the ticket: diameter 172: 232 Kg, diameter 216: 366 Kg.
- Banknote length: 3700 mm.
- Casting speed: 200 mm/min.

- Cooling water flow: 1100 l/min.
- Casting time: diameter 172: 41 minutes, diameter 216: 53 minutes

I.3.1.2. Biellets homogenization furnace

The charge is inserted on the carrier chassis into the heat chamber of the oven for homogenization by the horizontal circulation of hot air provided by circulation fans and a heating chamber provided at the top of the treatment chamber. The main characteristics of the homogenization unit are:

- Treatment load: 12000 kg/load.
- Processing temperature: 560 to 580 °C.
- Treatment time: 11 hours.

The introduced bills are then subjected to cooling formed by cooling fans provided on two sides, the hot air is evacuated from the chamber by a provided on the roof. The main characteristics of the cooling chamber are:

- Treatment capacity: 12000 Kg/load.
- Refrigeration range:
- 550 to 250°C: 170°C/h.
- 250 to 200°C : 100°C / h.
- 200 to 60°C : 56°C / h.

During the cutting of the bills of the copeaux are produced which are recycled in rework.

I.3.2. Extrusion workshop

Hot extrusion is a plastic deformation forming process that consists of inserting a heated block of aluminium metal or alloy of aluminum (billet) (350 to 500 °C) into a hollow, self-heated container, and, by means of a piston, pushing the bill with a pillar or fountain through the opening practiced in a forming tool (string) obturating the other end of the container. (**Figure I.2).**

The press summary is rigidly connected by columns to the press pot. The latter is supplied with water or oil, at pressures of up to 30 MPa, by a set of fixed and/or variable flow pumps that communicate to the piston and therefore to the footer a more or less rapid advance speed of 50 mm/s and more.

The chain, which gives the product the desired section, is supported on the press summary through a counter chain and other support valves.

The container is usually cylindrical but can be rectangular for the extrusion of wide profiles. It is constructed to withstand the pressures developed during extrusion and has its own heating device (by resistance or induction). It is heavily knotted against the chain by means of beams. Its inner diameter varies, according to the press, from 100 to 500 mm for bill lengths ranging from 400 to 1,200 mm [61].

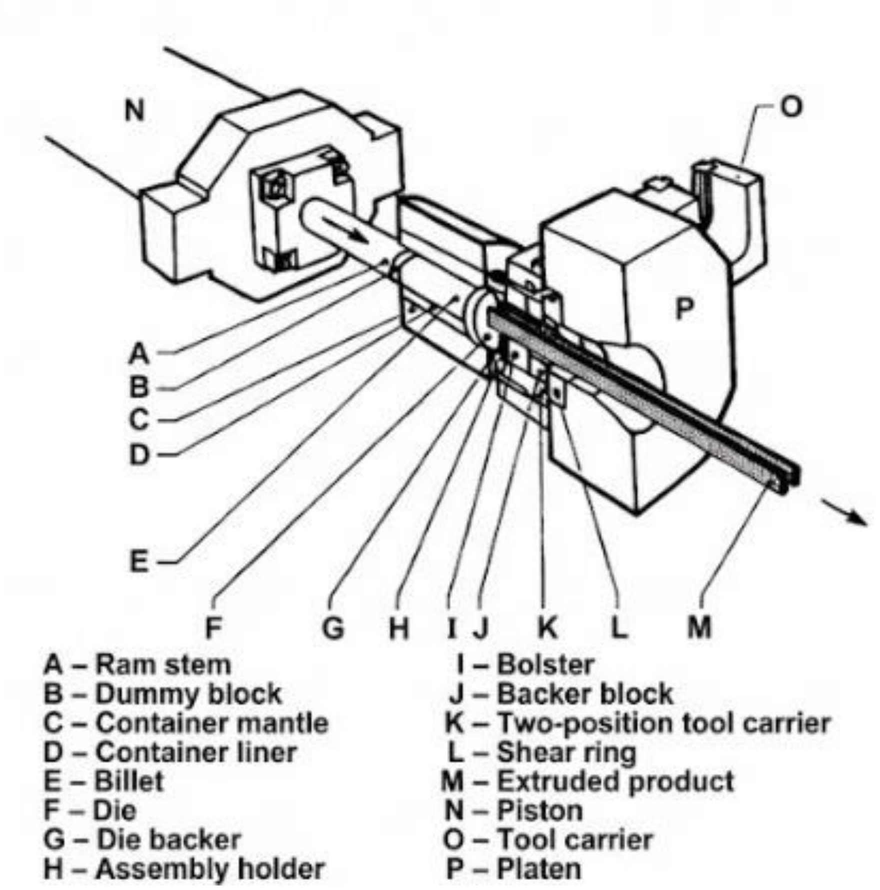


Figure I.2: *Main organs of a horizontal press [49].*

The equipment of the extrusion unit is:

I.3.2.1. Billet heating furnace

Billet are transferred from the loading table to the oven rollers. The fast-heating oven has a preheating area provided by fumes from the directly heated part of the oven. Through the oven, the billets reach temperatures of 430°C to 480°C (depending on the type of chain) and pass through the chassis frame and the hot scissor. Once sewn to a calculated length, they are transferred to the press. When this position is reached, the lingot push cylinder places the billet in the press loading.

I.3.2.2. Die oven

It is an electric furnace with hot air circulation intended for the preheating of the pipelines. Powered at a voltage of 380V and 50Hz.The wire oven has a power of 54KW and can reach a temperature of 450°C. Each press line has two chain furnaces. The regulator is automatically controlled by a thermocouple that measures the temperature in the oven..

I.3.2.3. Aluminum extrusion press

There are two types of presses to extrude:

- 1600 T press, powered by 172/178 billet diameter.
- 2500T press, powered by 216 billet diameter.

These are single-acting hydraulic extrusion presses of the horizontal type, with direct hydraulicly control allowing the pulse, cycle and synchronized operation of presses whose purpose is to wire aluminum alloy profiles.

Table I.1: Specifications of the two presses of the Algal plus plant

Specifications	**Press 1600**	**Press 2500**
Billet diameter	172	216
Die diameter	220	280
Max length billet racing	783	833
Main piston stroke	1850	2000
Container stroke	350	450
Container temperature	450	450
Spinning capacity	1632 T	2500 T

I.3.2.4. Aging Oven

Used to improve the mechanical characteristics of extruded profiles. The main characteristics of the aging oven are:

- Processing capacity: 6000 Kg Al/load.
- Maximum temperature: 300°C.
- Treatment temperature: 180°C.
- Treatment time: 4 to 8 hours (depending on the profile).
- Operating condition: 16 h/day.

I.3.2.5. Die correction workshop

The dies are the parts of extrusion spinning tools. Their design, development, manufacture and implementation require the utmost care. Until now, the design and development of chains can be regarded as an art primarily based on observation and experience, but CAD (computer-assisted design) can increasingly replace human empiricism and knowledge. After adjustment, it is necessary to harden the chain ranges, the process used is gas nitration which allows to obtain a diffusion layer of thickness less than or equal to 0.5 mm with a surface hardness ranging from 700 to 1200 HV and without risk of fragility. The wires must be perfectly adjusted before this nitridation because they are then too hard to be retouched (Figure I.3).

The different processes for die correction are as follows:

I.3.2.5.1. test spinning

Spinning a sample is carried out to check the spinning condition of the profiles or the difference when spinning with several flows.

I.3.2.5.2. Washing the dies

After the spinning operation, the die is dismantled from the tooling then immersed in a caustic soda solution whose concentration is 400 g/l and the temperature is 98°C for 5 hours in order to remove the deposit from it. aluminum of its scope.

I.3.2.5.3. Die scope polishing

If the die does not present any spinning defects, and does not require any correction, it will be subjected to polishing of its spinning surface in such a way that the profile will not present any surface defects.

I.3.2.5.4. Die correction

After the test spinning, if there are any defects, we proceed to the correction.

I.3.2.5.5. Liquid jet ironing

It is a preliminary nitriding treatment in order to reveal the surface of the metal for all scopes of the “ironing” sector. It is done with an abrasive liquid for two to three minutes.

I.3.2.5.6. Die scope polishing

After washing, the die seat must be polished to prepare it for the next spinning; first with No. 240 abrasive paper until you obtain a mirror surface.

I.3.2.5.7. Nitridation

The nitriding of the dies is carried out in a vertical oven by the addition of ammonia gas (NH3), the nitriding process is as follows:

- Increase in temperature to 560°C for 5 hours and injection of diatomic nitrogen extracted from the air from 200°C to make the environment neutral in the oven to avoid oxidation.
- Then we maintain the temperature at 560°C for 8 hours and inject ammonia NH3 and methanol (CH3OH) to make the medium rich in carbon and therefore avoid decarburization of the dies,
- Afterwards, we cool from 560°C to 200°C for 1 hour and 30 minutes in air using a fan.
- Finally, diatomic nitrogen is injected for 20 minutes.

I.3.2.5.8. Tooling assembly

The hollow type die is composed of the die and the porthole. The full type die is made up of the die and the counter die.

I.3.2.5.9. Storage

In order to prepare for their next spinning, the dies are stored and arranged in numerical order on the shelves near each press.

I.3.2.5.10. Spinning

The die is mounted in the die holder with the other elements in order to undergo reheating to 450°C in the die oven. The production quality of the dies depends on the material of the profiles, in general it must be replaced after ten to thirty profiles. The lifetime of the die depends on the nature of the profile, it will be rejected, generally when the number of nitriding executions exceeds six operations.

We can summarize the Circuit of a new Die by the following figure:.

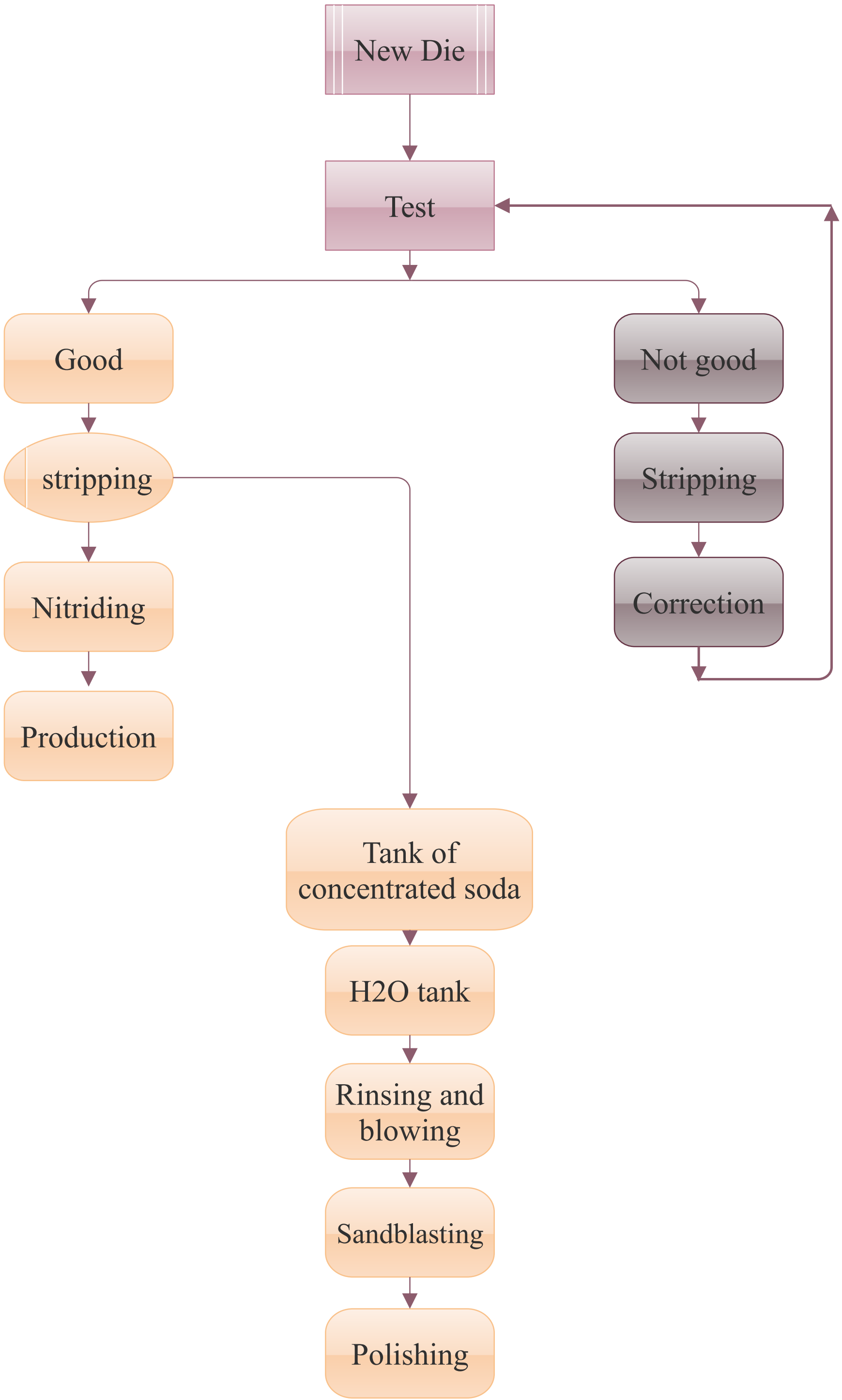


***Figure I.3**: Circuit of a new Die*

I.4. Conclusion

This chapter has allowed us to explore in detail hot working tools, focusing particularly on aluminum extrusion dies. An in-depth understanding of the failure modes affecting these critical components has been developed, highlighting the crucial importance of thermal fatigue.

Extrusion dies are subjected to extremely severe service conditions, involving repeated thermal cycles resulting from intermittent contact with hot metal and cooling systems. These temperature fluctuations induce cyclic thermal stresses in the die material, inevitably leading to the initiation and propagation of cracks through thermal fatigue, thus representing a major failure mode.

A critical review of the state of the art on the reliability of structures damaged by thermal fatigue has been presented, shedding light on recent advances as well as current challenges and limitations. Although significant progress has been made in understanding damage mechanisms and developing modeling by coupling rheological behavior models deduced from analysis results and prediction techniques, several scientific and technological challenges remain.

CHAPTER II

MECHANICAL RELIABILITY

Chapter II

II.1. Introduction

There exist two major models in design structuring process as basic structures. The first is according to a deterministic view and the second according to a probabilistic view.

The need to review some concept of the reliability-based method is felt since it helps to understand the approach for reliability analysis. The scope comprises design, operation, re-qualification and abandonment of the structure [62, 63].

It is necessary to ensure that the control of uncertainty is sufficient so that the risks are properly assessed and remain acceptable. This pertains to the field of reliability theories.

A formal explanation of what reliability means is given by AFNOR [64]: « *ability of a device to perform a required function under given conditions, for a given duration…the term is also used as a characteristic designating a probability of success or a percentage of success* ».

The difference highlights the relationship between a qualitative definition (aptitude) and a quantitative definition (probability). This concept promptly settles any potential disagreement by linking the management of uncertainty to probabilistic modelling. Furthermore, it advocates for the utilization of random variables and stochastic processes, albeit these are not exclusive methodologies; the techniques and instruments of fuzzy logic, convex sets, and resilience may all provide valuable contributions. [64].

However, this approach allows you to clearly understand the risks taken. In the analysis of the reliability of structures, the influential parameters are recognized as probabilised variables and the probability of failure and inspections, the residual life, etc. are calculated. Feedback and knowledge of kinetics degradation are two of the essential conditions for applying structural reliability analysis.

Structures age, the properties of materials may be altered, operating methods are no longer the same.

The probabilistic approach to reliability of structures then proves essential, the risk is evaluated in the form of a probability and no longer in the form of a binary judgment (the design is acceptable or not, the operation can be continued or not).

Calculating this probability makes it possible to reduce the risk of failure by organizing maintenance and inspection programs and to extend the operating life by optimizing their use.

II.2. Reliability analysis of the mechanical structures

A detailed study on reliability methods was developed by Lemaire [64]. We will also find an important historical reminder of mechanical reliability in an in-depth synthesis described by Richard [65], the philosophy associated with this approach, its challenges as well as the detailed explanation of the calculation methods. We propose here to briefly recall the main principles and the different stages of such an analysis necessary in the case of random phenomena independent of time.

A significant number of manufacturers consider that the safety offered by a structure is considered sufficient, until a sudden anomaly would expose people, property and the environment to harmful danger. The structure reliability is defined by its performance in fulfilling a defined function under given conditions, for a fixed duration and while respecting the required level of safety.

The difficulty encountered in evaluating the reliability of a structure comes essentially from the uncertainty of the phenomena involved in the structure considered (uncertainties of the applied loads, variabilities in the properties of the materials, geometric inaccuracies, etc.), which will have a direct influence on system performance. Also, it is necessary to integrate these uncertainties in order to work on realistic modeling.

Traditionally, the deterministic approach takes all of the aforementioned parameters with a fixed value, to design the dimensioning of structures. Particularly, the uncertain parameters are given by unfavorable characteristic values, linked to safety coefficients; we then find ourselves with a binary result (“**safety**” or “**failure**”) with respect to a given criterion, reflecting the confidence that can be allocated to this precise dimensioning. Inevitably, an unjustified oversizing will be obtained, via the deterministic approach which uses a deliberately pessimistic margin.

Although, in the probabilistic approach, we use stochastic modeling in which uncertain data are represented by random variables, we can then evaluate the probability of failure of the structure, while measuring its sensitivity in relation to each of the random variables introduced.

In the literature, two applications are possible: either we know the characteristics of the structure, by verifying its reliability that it is satisfactory; or we optimize its dimensioning so as to respect a given level of reliability. Therefore, the reliability approach allows a better assessment of safety margins using objective confidence indicators, and in this sense constitutes an adequate tool for decision support in the design and maintenance phases, through a more rational treatment of uncertainties.

We then focus on the reliability of a mechanical system (also called structure). In this context, the reliability analysis is broken down into three stages:

a- The selection of basic random variables, which integrate the uncertainties involved via their distribution laws;

b- The choice of a performance function which defines the failure of the structure;

c- The calculation of reliability indicators, which provide a quantitative and qualitative assessment of the reliability of the structure.

II.3. Dimensional model

The validation of equality as a function of time T is indicative of the efficacy of dimensioning.

The most basic formulation involves treating the resistance R and the acting stress S as time-independent output variables. The structure's state can be described using a single global random variable, the margin, which is the simplest example of resistance-solicitation.

II.4. Principles of structural reliability

II.4.1. Failure probability

The operating scenario is characterized by the presence of an increased resistance to the stress., i.e.:

$$G(r,s) = r - s > 0 \tag{II.1}$$

and the scenario of non-operation, or failure, is then:

$$G(r,s) = r - s \leq 0 \tag{II.2}$$

The indicator of failure is resulting in the probability correlated with the event. $\{R - S \leq 0\}$;

Failure Probability:

$$P_f = \Pr ob(\{R - S \leq 0\}) \tag{II.3}$$

Reliability is, for its part, described as the complement of the failure probability:

$$\text{Reliability} = 1 - P_f \tag{II.4}$$

R and *S* are two random variables characterized by a joint probability density noted

$f_{R,S}(r,s)$.

If the risks taken stay within tight geometric and chronological bounds, a lower level is quite reasonable. **Table II.2**, often cited in documents concerning civil constructions, proposes objective values according to certain construction situations, without however clearly specifying the reference durations.

Table II.2 *Order of magnitude of target probabilities in construction*

Average number of people put at risk	**Economic consequences**		
	No risk	**risk**	**High risk**
Little (<0,1)	10^{-3}	10^{-4}	10^{-5}
Medium	10^{-4}	10^{-5}	10^{-6}
Big (>0,1)	10^{-5}	10^{-6}	10^{-7}

Even its evaluation is extremely computationally expensive, because it is a very small quantity and because not all the necessary information on the joint probability density is available. For these reasons, more efficient methods have been developed, they are known as *FORM/SORM (First/Second Order Reliability Methods)*. These techniques follow an approximation of the probability of failure after the computation of a given measure of reliability known as reliability index. The method of dependability calculation is shown in the flowchart (**figure II.4**). Defining the mechanical and probabilistic models to be applied in the study comes first once the current knowledge about the system is obtained. The specification of possible failure situations complements these two models to generate what we refer to as the mechano-reliable model. Monte Carlo simulation methods can be applied in the absence of direct integration of the very rare probability of failure provided the cost of the mechanical calculations keeps low.

The numerical calculation of the reliability index is a useful method for analyzing industrial systems in practical scenarios. The design point is derived from a specific optimization process, enabling direct or indirect control of the mechanical model. At the conclusion of this phase, the required probability is assessed using either the FORM/SORM methodologies or by conducting importance sampling around the design point.

Regarding to the reliability level, the process enables the designer to evaluate the significance factors for various mechanical and probabilistic elements. This information is crucial for system optimisation, including uncertainties at various stages of design, production, installation, and maintenance.

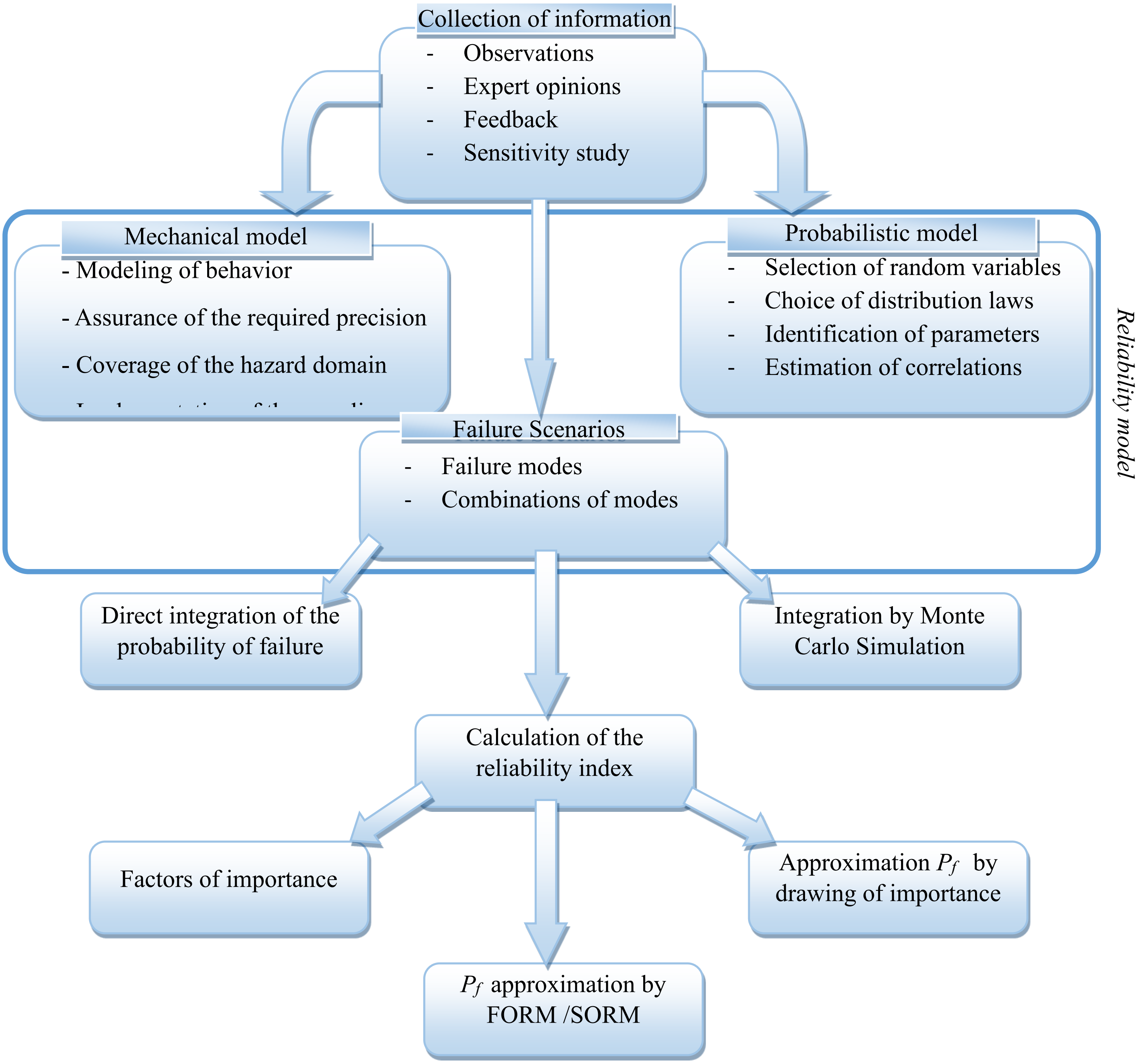


Figure II.4 *Flowchart of reliability analysis of mechanical systems*

II.5. Variabilities description

When the control of physical properties leads to the acquisition of measurements that cannot be strictly imitated, it is necessary to define a device representative of the variabilities recorded for the calculation of structures and whose formulation adjusted for integration into simulation codes must be called through popular models. So, a modeling of this various information via mathematical laws which will offer the frequencies of occurrence of each result, their average value and the swings around this average, becomes very necessary at this stage.

In order to work on a significant set of information, distribution modeling must have a sufficient number of data points. Recognizing that a real and continuous random variable *X* is known to have a specific number *N* of independent values (or results) $\{xi\}_{i\in[1..N]}$ of *X*, which constitutes a sample. First and foremost, the statistical treatment of this data frame enables the analysis of its variability: the distribution of values (using relative and cumulative frequencies, etc.) as well as the characteristics of central tendency (mean, ….) and dispersion (type of deviation, ...) [66].

Therefore, the contribution of uncertainty modeling, as observed through mathematical functions that distribute issues for *X* and assign each issue its probability of occurrence, is highly beneficial at this stage. In this scenario, one may refer to the probability density function f_X : R → R or the probability partition function F_X : R → [0, 1] associated with *X*, such as (*P* the probability):

$$f_X(x)dx = P(x \leq X < x + dx)$$
$$F_X(x)dx = P(X < x)$$

(II.5)

With the links:

$$\forall,\ F_X(x) = \int_{-\infty}^{x} f_X\,(u)du \quad et\, \frac{dF_X}{dx}(x) = f_X(x) \qquad \text{(II.6)}$$

And checking:

$$\forall x, \qquad f_X(x) \geq 0$$

$F_X(x)\ is\ non-decreasing$

$$F_X(-\infty) = 0,\ F_X(+\infty) = \int_{-\infty}^{+\infty} f_X\,(u)du \qquad \text{(II.7)}$$

Graphically resembles the area under the curve of f_X between x = -∞ and x = a, then the probability that X is between two given values a and b, is given as follows :

$$P(a \leq X < b) = F_X(b) - F_X(a) = \int_b^a f_X\,(u)du \qquad \text{(II.8)}$$

Showing the part under the curve *fX* between x = a and x = b (**figure II.5**). The probability of an event being naturally linked to the notion of statistical frequency for large samples, we therefore determine by an adjustment procedure the functions *fx* and F which best represents the experimental variabilities.

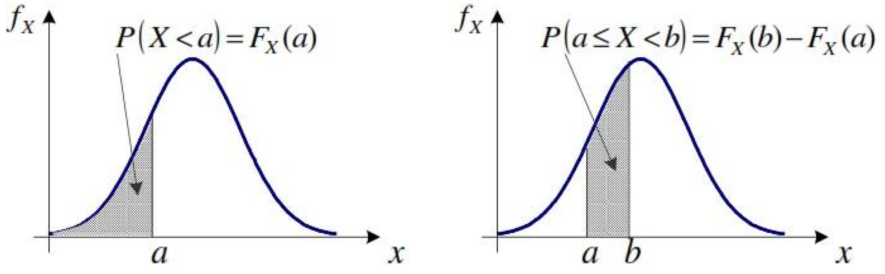


Figure II.5 Graphical interpretation of the density and distribution functions of a random variable X [67].

In the case of reliability analyses, we generally take more than one random parameter. The set of *k* variables X_i of the problem forms in this case a vector $\{X\} = \{X_i\}_{i=1..k}$ of R^k whose appearance probabilities are described through a combined of probability density.

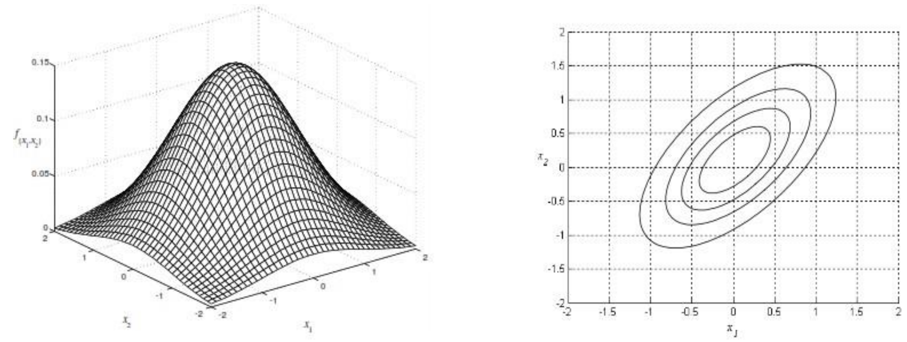
$f_{\{X\}}$: $R^k \rightarrow R$ and an integrated of distribution function $F_{\{X\}}$: $R^k \rightarrow [0, 1]$ as:

$$f_{\{x\}}(\{x\}) \prod_{i=1}^{k} dx_i = P(x_i \leq X_i < x_i + dx_i), \forall_i = 1..k \qquad \text{(II.9)}$$

or $\{x\} = \{x_i\}_{i=1..k}$ is a vector of R^k, with :

$$\int_{R^k}^{P} f_{\{x\}}(\{x\}) \prod_{i=1}^{k} dx_i = 1 \qquad \text{(II.10)}$$

(Figure II.6) illustrates the appearance of a function $f_{\{X\}}$ In the scenario containing two random variables.



(a) (b) : $f_{\{X_1,X_2\}}(x_1,x_2) = \text{cste}$

Figure II.6 Joint probability density function in the case of two random variables {X} = {X1, X2} : (a) representation 3D, (b) isovalues of f{X} [68].

When the *k* random variables X_i are independent, the expression of the joint density reduces to the product of the marginal densities f_{Xi} of each variable:

$$f_{\{x\}}(\{x\}) \prod_{i=1}^{k} f_{x_i}(x_i), \forall \{x_i\}_{i=1..k} = 1 \qquad \text{(II.11)}$$

Otherwise, it is necessary to absolutely specify the nature of their dependence for the calculation of reliability indicators. This is why a correlation matrix is introduced $[\rho] = [\rho_{ij}]_{(i,j)=[1..k]^2}$ whose components ρ_{ij} measure the linear dependence between each pair of random variables X_i and X_j as follows:

$$\rho_{ij} = \rho(X_i, X_j) = \frac{cov(X_i, X_j)}{s_{X_i} s_{X_j}}, \forall (i,j) \in [1..K]^2 \qquad \text{(II.12)}$$

or *Cov* (X_i,X_j) represents the covariance of these two variables, and designating their respective standard deviations (see for example Jolion [67]). Each component ρ_{ij} is a number belonging to the interval [−1,+1]:

1- if $\rho_{ij} = 0$, the variables X_i and X_j are not linearly linked to each other (but can be in another way);

2- if $\rho_{ij} > 0$, X_i and X_j tend to vary in the same direction;

when $|\rho_{ij}|>0.9$, the linear connection is considered strong Let us specify, finally, that for variables $\{X_i\}_{i=1..k}$ independent, the correlation matrix is reduced to the order identity matrix *k*.

3- independent, the correlation matrix is reduced to the order identity matrix.

II.6 .Reliability calculations methods:

II.6.1. Calculation of failure probabilities:

A component's failure probability, denoted as P_f, is determined by the following formula [69]:

$$P_f = \int_{G(x_i)\le 0} f_{x_1,x_2,\dots,x_n}(x_1,x_2,\dots,x_n)dx_1dx_2\dots dx_n \tag{II.13}$$

In this context, x_i represents a vector of stochastic variables, $f(x_i)$ denotes the joint probability density function, and $G(x_i)$ is the limit state function, where $G(x_i)<0$ indicates failure. The function $G(x_i)$ may denote a singular failure cause, i.e., a single event function, or a system representation of multiple failure modes. Only a limited number of analytical solutions for the aforementioned integral exist, and conventional numerical integration is often time-consuming and expensive due to the numerous stochastic variables typically present in dependability applications. Estimates of the failure probability can be derived by complementary methods:

- Formal or numerical integration [70].
- Monte Carlo simulations [71].
- Approximate analytical methods [70].

In the past two decades, there has been a substantial advancement in the development of structural reliability theories and methods. Improvements have been made to computation methods and philosophical and conceptual issues through extensive research. The methodology that has been devised is now being widely implemented in the field. The methods and application of structural reliability theory have been extensively documented [70,72], and there are numerous papers that address the subject[72,73]. A number of analysis tools are at the present time available.

II.6.2. Nested reliability method:

This is required in case of inclusion of reliability calculations at the limit state function, i.e. in case of :

- Applications involving time-dependent variables conditioned on time-independent variables.
- First passage failure event as out-crossing of a stochastic process through an uncertain failure surfaces.
- Determination of the distribution of the failure probability due to subjective uncertainties.
- Separation of differentiable and non-differentiable variables. The non-differentiable variables (e.g., discrete variables) have to be analyzed by simulation techniques.

In time-dependent problems, emphasis is on the study of the probability that a stochastic vector process (e.g. a load process) leaves a safe domain at least once within the time interval, can be modeled as a stochastic variable with a given (extreme value) distribution function. For several, time-dependent load processes, the extreme distribution value is in general unknown, and a useful concept for determination of the first passage failure probability is the mean rate of out-crossing of a vector process from the safe region. The first passage failure probability can be approximately given in many cases, e.g. for high thresholds and not too narrow-banded process. This method is valid for stationary conditions otherwise it is very difficult and even impossible to use.

II.6.3 The Monte-Carlo Method:

The basic Monte-Carlo simulation method samples from the joint distribution $f_x(x)$, and the indicator function $I(x)$, defined as:

$$I(x) = 1 \text{ if } G(x) \leq 0 \quad \text{and} \quad I(x) = 0 \text{ if } G(x) > 0, \tag{II.14}$$

is evaluated at each sample point. An unbiased estimator for the failure probability is then given by the sample mean in the form:

$$\hat{P}_f = \sum_{i=1}^{N} I(x_i) \tag{II.15}$$

An advantage of this method is that it makes use of point values of *G(x)* only, and the distributions do not require any analytical properties. The disadvantage is the computational time for small probabilities. If n is the number of simulations, it is admitted that, the failure events frequency tends towards the probability of failure when n → ∞:

$$P_f = \lim_{n \to \infty} \frac{number of\ events G \leq 0}{total number of\ simulated events} \tag{II.16}$$

II.6.4. Approximate analytical methods:

Approximate asymptotic methods include the so-called first order reliability method (FORM) and second order reliability method (SORM). The basic approach of the analytically based approximate methods is a transformation, of the stochastic variables into probability space of standardized, mutually independent variables, and to approximate the probability integral by simplifying the boundary of the integration domain, appendix A.

These methods have support from an asymptotic analysis and are particularly useful for the high reliability problems often encountered in engineering. A good approximation to the failure probability can often be obtained by replacing the true boundary of the integration domain by the first order (FORM) or second order (SORM) approximations In this case *G(x)* is twice continuously differentiable in the vicinity of the designed point. The simplified approaches are driven by the fact that, in some situations, computing points on the failure surface is quite expensive; e.g., if for every iteration in normalised space a fresh time-consuming finite element analysis are needed. Therefore, it is crucial to define strategies whereby minimum *G(x)* computations are maintained. It should be noted that application of response surface approaches could sometimes help to reduce the amount of iterations. Thus, the complement of the likelihood of failure defines the dependability R(t).:

$$R(t) = 1 - P_f. \tag{II.17}$$

II.7. Reliability indicator

The following is the conventional concept of a structure's reliability R: [64,82]:

$$R=1-P_f \qquad \text{(II.18)}$$

This is the case where the probability of failure, denoted by Pf, is equivalent to the probability of the event occurring. $G(\{X\}) \leq 0$:

$$Pf=P(G(\{X\}) \leq 0) \qquad \text{(II.19)}$$

This indicator of confidence is the most important result that may be obtained from a calculation that is reliable. In the event that we incorporate the combination of probability density $f_{\{X\}}$ random variables $\{X\}$ into the model, the probability of failure can be expressed in the following manner:

$$P_f \int_{R^k}^{P} f_{\{x\}}(\{x\}) \prod_{i=1}^{k} dx_i \qquad \text{(II.20)}$$

This integral can only be investigated analytically or even numerically very infrequently. It is extremely limited in scope. In the real world, we do not typically have access to the combined probability density of the vector {X}. Instead, we are typically required to be content with the minimal rules that pertain to each variable Xi and some information regarding their correlation. There is also the possibility of defining the integration domain Df in an implicit and complex manner in accordance with the mechanical model that is kept.

Consequently, a number of different resolution strategies have been devised in order to solve these challenges [74,75]. Methods that are based on simulations and those that use an approximation are the two primary categories that are considered to be the most traditional.

II.8. Reliability index

The principle of the Cornell reliability index (or simply: *Cornell index*) consists of measuring the distance between the average point of the margin $G(x_i)$ and the point where the margin becomes zero (failure point), this distance is measured in number of standard deviations. In other words, to evaluate reliability, we determine how many standard deviations separate the average operating state from that of failure. If the mean and the standard deviation of the margin G are denoted by m_G and σ_G, respectively, as shown in (**figure II.7**), then the Cornell index β_C can be expressed as follows:

$$\beta_C = \frac{m_G}{\sigma_G} \tag{II.21}$$

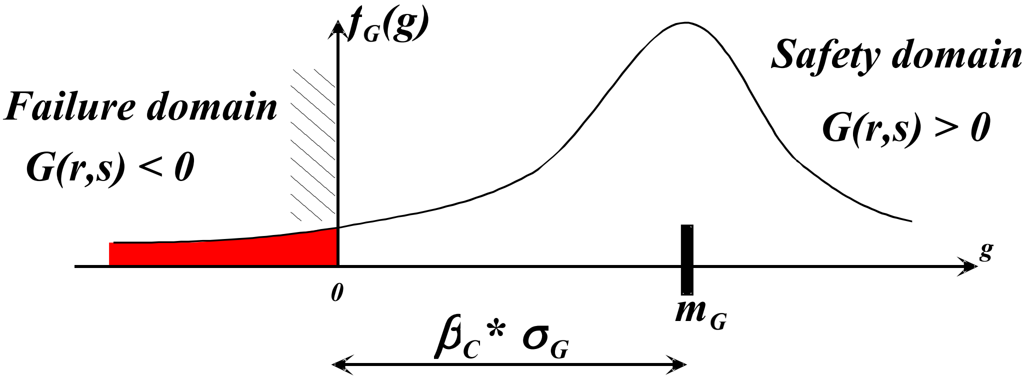


Figure II.7: Reliability margin and Cornell index

If the distribution of the margin follows a normal law, As a direct result of the following, the probability of failure is:

$$P_f = \Phi\left(-\beta_C\right) \tag{II.22}$$

where $\Phi(.)$ is the standardized normal distribution law. The error in this expression increases as the margin deviates from the normal distribution.

To illustrate this index, Let us consider the scenario of two Gaussian variables, R and S, that are independent and have means m_R and m_S, and standard deviations σ_R and σ_S, respectively. The safety margin and its parameters are given by :

$$\begin{gathered} G = R - S \\ m_G = m_R - m_S \\ \sigma_G = \sqrt{\sigma_R^2 + \sigma_S^2} \end{gathered} \tag{II.23}$$

The Cornell index takes the form:

$$\beta_C = \frac{m_R - m_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{II.24}$$

In this case, the probability $P_f = \Phi\left(-\beta_C\right)$ is exact because G law is also Gaussian.

Let us now consider the case of the same limit state written in another form:

$$G = 1 - \frac{S}{R} \tag{II.25}$$

he law of G is no longer normal and the relation $P_f \approx \Phi\left(-\beta_C\right)$ becomes more or less approximate. In this case, the Cornell index only gives a measure of security, which is not directly related to the probability of system failure.

With a view to providing a representative and invariant measure of reliability, Hasofer and Lind [75] proposed not to place oneself in the space of physical variables but to carry out a change of variables and thus place oneself in the space of physical variables. space of reduced centered Gaussian variables (i.e. zero means, unit standard deviations) and statistically independent. Transformation of variables x_j in normalized variables u_i is written by:

$$u_i = T_i(x_j) \tag{II.26}$$

This transformation is called *isoprobabilist transformation*, it is illustrated in Figure III.7. In this new space, the limit state function takes the form:

$$G(x_i) = G(T_i^{-1}(u_j)) \equiv H(u_j) = 0 \tag{II.27}$$

The probability of failure is expressed in writing:

$$P_f = \int_{H(U)\leq 0} \phi_n(u)\, du_1 \cdots du_n \tag{II.28}$$

Where $\phi_n(u)$ is the density function of the centered normal law reduced to n dimensions.

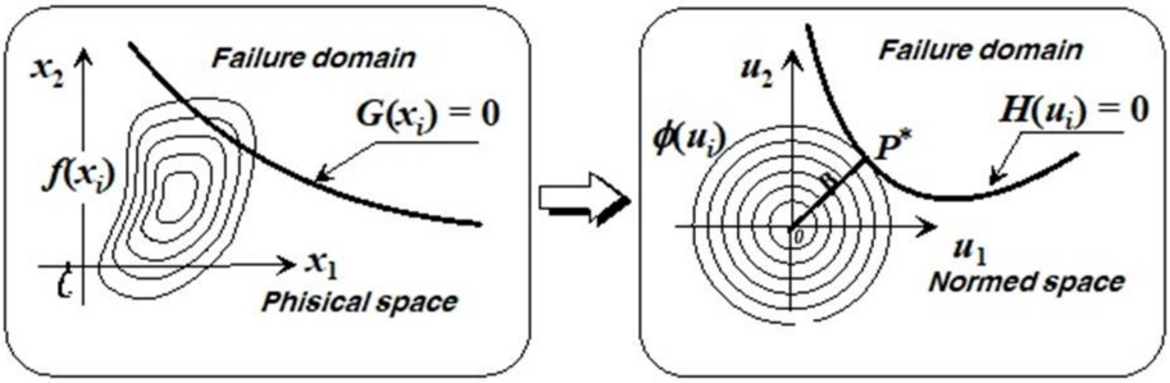


Figure II.8 :Isoprobabilistic transformation [34].

The mathematical model of Hasofer and Lind states that in the normal space reliability index β is the minimal distance between the origin and the failure domain $H(u_i) \leq 0$. This distance specifies a hyperplane tangent to the limit state function and a location, sometimes known as the most probable design point or point of failure (**figure II.8**).

Find β is therefore a constrained optimization problem:

$$\beta = \min_{\{u\}} d\left(\{u\}\right) = \sqrt{\{u\}^t\{u\}}$$

under duress : $H(\{u\}) \leq 0$ (II.29)

The search for the point P^* can be carried out by an optimization method adapted to the particular form of the problem.

A first approximation of P_f is obtained by replacing the limit state $H(u_i) = 0$ b y a hyperplane tangent to the design point P^*; this is the First Order Reliability Method, FORM. Taking into account the rotational symmetry property of the standard probability density, we can estimate this probability by [76,77,78]:

$$P_f \approx \Phi\left(-\beta\right) \tag{II.30}$$

where $\Phi\left(\cdot\right)$ is the Gaussian distribution function. The degree of precision of this approximation depends on the nonlinearity of the limit state.

To determine the importance of the variables, we need to evaluate the direction cosines of the unit vector normal to the performance function in the norm space. The weight of each of the standardized random variables in the evaluation of the reliability index is written:

$$\alpha_i = -\left.\frac{\partial \beta}{\partial u_i}\right|_{P^*} = -\frac{u_i^*}{\beta} \tag{II.31}$$

where α_i is the direction cosine of the variable u_i and P^* is the design point.

II.9. Treatment of uncertainties

The basic variables involved in the failure equation are physical variables:

- Geometry;
- The properties of the materials, in particular the degradation kinetics, the coefficients of the degradation laws, the elastic limit and the breaking strength;
- Maintenance and inspection methods and their effectiveness.

We must not deprive ourselves of looking for similar structures or installations, under the same operating – maintenance environmental conditions.

As with any operational safety problem, it is necessary to check:

- The accuracy of the data.
- Their representativeness.
- Their relevance.

These variables are, moreover, uncertain and all uncertainties must be considered in the case of a reliability analysis.

Basic variables with little uncertainty are considered decisive. For the other variables, it will be appropriate to present them by a histogram or by problem distribution.

We can distinguish different types of uncertainties:

- Uncertainty with operating time;
- Uncertainty in space;
- Errors due to measuring instruments;
- Statistical uncertainty due to the number of observations of a basic variable;
- Finally, the uncertainty due to the reliability model of the structures used, to the equation of

failure used to explain physical phenomena and their effects. This latter uncertainty is reducible by an increase in knowledge;

For better reliability assessments, it is necessary to always reduce uncertainty, in particular by supplementing the samples of the basic variables:

- By collecting even more data, particularly those which prove necessary;
- By collecting similar feedback;
- By enriching the data with expertise, even if this expertise is vague and dispersed;
- Using statistical methods.

II.10. Lifetime prediction

Modeling temporal variabilities uses the notion of stochastic processes. These processes are assimilated to an infinity of random variables indexed on the coordinate of the deterministic parameter considered (space or time). In addition to the mean and the standard deviation which can evolve with the indexing variable, stochastic processes are characterized by the auto-correlation function, which implies a certain dependence between neighboring points of the same process.

In a moment t of the life of the structure, the probability of instantaneous failure is:

$$P_f(t) = \Pr\left[R(t) \leq S(t)\right] \tag{II.32}$$

If the instantaneous densities $f_R(x,t)$ and $f_S(x,t)$ are known, the probability of instantaneous failure $P_f(t)$ can be calculated by the convolution integral:

$$P_f(t) = \int_{-\infty}^{+\infty} F_R(x,t)\, f_S(x,t)\, dx \tag{II.33}$$

An alternative method involves analyzing the progression of the safety margin G(t) throughout the structure's lifetime, wherein the margin is represented as a stochastic process. Consequently, we aim to determine the probability that the margin is either negative or zero during the observation interval; this methodology is referred to as the crossing method (**figure II.9**). The moment when the margin first reaches 0 or turns negative is referred to as the time to failure, making it a random variable. The associated probability $\Pr\left[G(t) \leq 0\right]$ is referred to as the probability of the first crossing; for non-repairable systems, this equates to the probability of ruin.

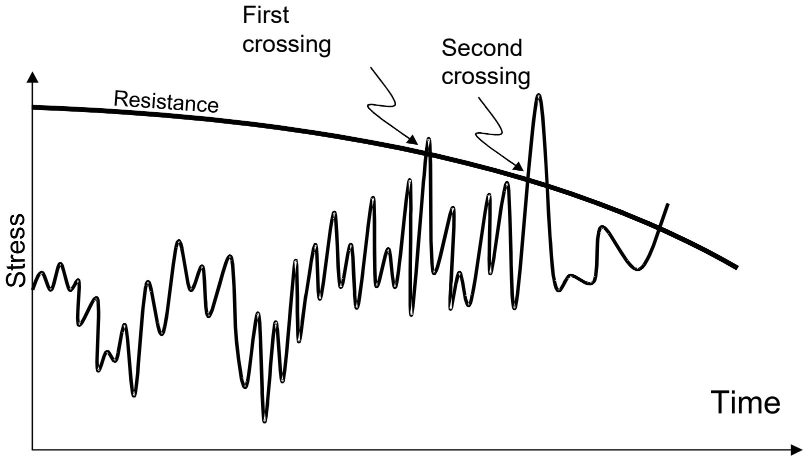


Figure II.*9* lifetime prediction

II.11. Reliability sensitivity analysis

The mechanical model facilitates the transfer from input data (fundamental variables) to output variables (**Figure II.10**). The objective is to compute the statistical parameters of the output variables based on the statistical parameters of the input data. This approach serves as a reliable sensitivity assessment, delineating the response's sensitivity to input variability. A first-order deterministic sensitivity analysis involves computing the gradient at a certain point, whereas a random sensitivity analysis examines the correlation between the coefficients of variation of an output variable and an input variable. entry. Two primary methodologies are employed [79]: the Monte Carlo approach, which operates through simulation, and the disturbance method, which necessitates the computation of the derivatives of the rigidity matrix and the vector of external forces concerning stochastic data. The Monte Carlo approach generates a sample from which statistical moments can be derived without prior constraints on order.

Usually, the disturbance technique spans only the first two moments. Constructing the solution becomes much more challenging in the case of an internal hazard on the state parameters of the mechanical system and behaviors, the solution is rather straightforward when the (external) hazard just concerns the actions and the model is linear. not linear. Usually conducted around the average operational point, a sensitivity analysis is not around a particularly interesting point. It clarifies whether the mechanical model damps or amplifies the unpredictability of data, therefore affecting the risks of instability involved. At least if it is Gaussian and stationary, the techniques of stochastic dynamics are now effectively fit in a dynamic environment to the analysis of the response process of a defined system aroused by an input process. Sensitivity analysis in the application of simulation depends on the capacity to build synthetic statistical samples of the data (quality of the random number generator, stochastic process generator) and on the capacity to recognize the samples of the variables or exit processes.

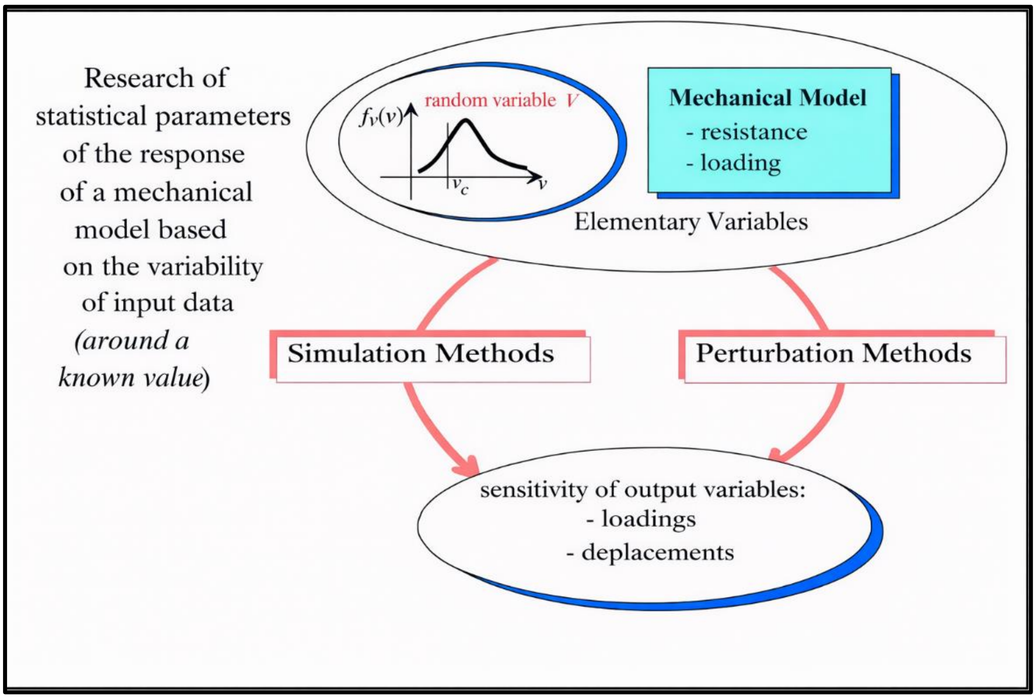


Figure II.10 *:Reliability sensitivity analysis*

II.12. Complexity of the mechano-reliability coupling

A mechanical-reliability model is the one that combines a mechanical calculation method with a reliability calculation method. Four criteria describe the relative complexity of the linkage of a mechanical model and a reliability model depending on the modalities they adopt (**figure II.11**):

The main elements to be investigated are the complicated of the mechano-reliability relationship. First, the nature of hazards needs to be considered. The second is the influence of time, which presents itself through static contexts, cyclical or dynamic conditions, and aging effects based on mechanical wear and physico-chemical degradation. Third, there is the mechanical model, which is a representation of the system being studied. This can be done using either simple linear elastic calculations or more complex elastoplastic calculations. These can be done using linearised sequences, limit state theorems, explicit or implicit nonlinear formulations, as well as geometric and material nonlinear calculations. Depending on the context, dynamic calculations can be also either linear or nonlinear. Representation of these intricacies continues to take the form of a performance function in which the variables of stress and resistance, explicit and linear random variable formulation, and regularity in various measures are separated. Depending on the nature of the problem, performance functions can have strong curvatures or singular points, explicit or implicit expressions. They represent the culminating complex interaction of mechanical behavior with reliability concerns in the mechano-reliability systems

Simple problems including external hazards, linear statics, explicit resistance-load function are very easy to resolve, whereas those leading to implicit functions in non-linear models will require very significant calculation resources. It is the issue of the risk incurred which will decide the precision of the modeling and the means to be implemented.

While those leading to implicit functions in non-linear models will require very large computation resources, simple problems involving external hazards, linear statics, explicit resistance-load function are quite cheap to tackle. The question of the risk involved will determine the means to be used and the level of modelling accuracy.

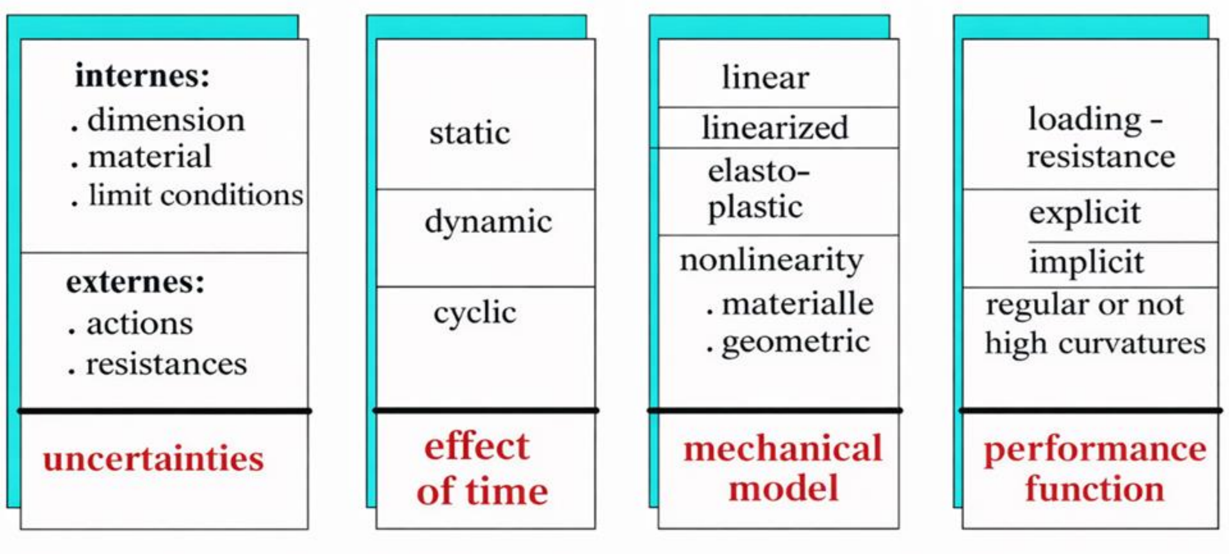


Figure II.11 : *Complexity of the mechano-reliability coupling*

Even if we will not use it in the rest of this work, we should nevertheless point out that there are also important simulation (or drawing) methods which consist of carrying out a reduced number of but targeted drawings, by occurrence around the design point [79]. This method therefore requires previously carrying out simulations of the isoprobabilist transformation then the optimization process for access to β.

II.13. Application to extrusion dies

In today's manufacturing industry, aluminum extrusion is a widely used process for producing complex shapes and profiles. One of the key factors in the aluminum extrusion process is the design and performance of the extrusion dies. The shape and dimensions of the die directly influence the final product's characteristics, making it crucial to understand the probabilistic aspects of die performance [80].

To study the probabilistic nature of aluminum extrusion dies, researchers have employed various techniques such as finite element analysis and Monte Carlo simulations. These methods allow engineers to assess the variability and uncertainty in die performance, considering factors like material properties, temperature distribution, and die geometry [81].

By quantifying the probabilistic behavior of extrusion dies, manufacturers can make informed decisions to improve product quality, optimize die design, and enhance overall process efficiency. This probabilistic study not only contributes to the advancement of aluminum extrusion technology but also holds significant potential for application in other manufacturing processes. [82,83,84] Additionally, this research can provide insights into the reliability and robustness of aluminum extrusion dies under different operating conditions.

Enhancing the reliability and efficiency of aluminum extrusion is of utmost importance. A study conducted by multiple authors highlighted the importance of implementing effective die design and optimization techniques in order to minimize faults and enhance overall productivity. Assaad's work emphasized the need of regulating the aluminum flow and enhancing die design to reduce flaws and enhance the quality of the product [38].

Sheikh et al. (2004) [80] conducted a study that examined the probabilistic patterns in the lifetime of tooling used in commercial aluminum extrusion. The study examined 595 instances of die failures, encompassing 17 distinct die profiles, in order to investigate the correlation between die reliability and profile complexity. The researchers employed different probability distributions to represent the lifetime of the die. They determined that the normal distribution was an appropriate match for the data, which encompassed instances of thermal fatigue failures. This study emphasizes the significance of comprehending the probabilistic characteristics of die failures in order to enhance die design and dependability.

II.14 Conclusion

As we continue to explore the frontiers of science and technology, the probabilistic study framework outlined in this chapter provides a powerful toolkit for addressing the inherent uncertainties and variabilities that arise in complex systems. By fostering a probabilistic mindset and integrating uncertainty quantification into the core of our research and development efforts, we can drive innovation, mitigate risks, and achieve unprecedented levels of performance and reliability.

The probabilistic framework developed in this research has proven invaluable in identifying critical sources of uncertainty and quantifying their impact on output quantities of interest. The ability to perform sensitivity analyses and uncertainty propagation has facilitated the prioritization of risk mitigation efforts, ensuring that resources are allocated effectively to address the most influential factors.

Furthermore, the integration of probabilistic methods with advanced optimization algorithms has enabled the development of robust and resilient designs that are less susceptible to variations in input parameters. This synergistic approach has the potential to significantly reduce costly iterations in the design and development processes, thereby improving overall productivity and cost-effectiveness.

CHAPTER III

LINEAR MECHANICS OF FRACTURE AND DIE DEGRADATION

Chapter III

III.1. Introduction

Extrusion dies are subjected to severe degradation mechanisms during operation, which can lead to mechanical and thermal damage, ultimately compromising their performance and service life. Mechanical damage can manifest in various forms, including abrasive wear, erosion, and plastic deformation. These mechanisms are primarily influenced by the high pressures, extrusion velocities, and frictional conditions at the die-billet interface.

Abrasive wear, caused by the relative motion between the die surface and the abrasive particles present in the aluminum billet or lubricant, can lead to material removal and surface roughening. Erosion, on the other hand, results from the impingement of hard particles carried by the high-velocity aluminum stream, causing localized material loss and surface degradation.

Additionally, the die undergoes significant plastic deformation due to the high extrusion pressures and elevated temperatures. This can lead to dimensional instabilities, distortion of the die geometry, and potential failure due to excessive deformation.

The temperatures another critical degradation mechanism affecting extrusion dies. During the extrusion process, the die experiences cyclic thermal loading as it repeatedly comes into contact with the hot aluminum billet.

III.2. Fracture mechanics

Scientific research carried out to understand and prevent accidents led to the birth of fracture mechanics, which aims to study the propagation of pre-existing macroscopic cracks within a material [85, 32]. It is a complex phenomenon which conditions, in many cases, the lifetime of structures. In reality crack propagation is a random process which depends on many uncertain parameters such as the initiation conditions, the initial length, the properties of the material, the geometry and dimensions of the structure, the intensity and nature of the loading at course of time and other factors inherent to the application.

The propagation of a crack in a given structure was at the beginning of the century and until the 1960s one of the major causes of catastrophic failures. The concern to secure structures and prevent catastrophic failures through a judicious choice of material and design is in fact old and continues to arouse the same interests [33].

Griffith [34] introduced in the 1920s the foundation of the theory of fracture mechanics, the aim of which is to predict the evolution of cracks until the structure in which they are located ruptures. He addressed the problem of the rupture of a cracked fragile elastic medium from an energetic point of view and showed that the rupture can be characterized by the rate of energy restitution, the critical value of which is an intrinsic characteristic of the material . **(Figure III.12)** compares the classic approach for the design of structures based on the elastic limit of the material σ_E to the approach using the concept of toughness K_C from linear fracture mechanics(MLR).

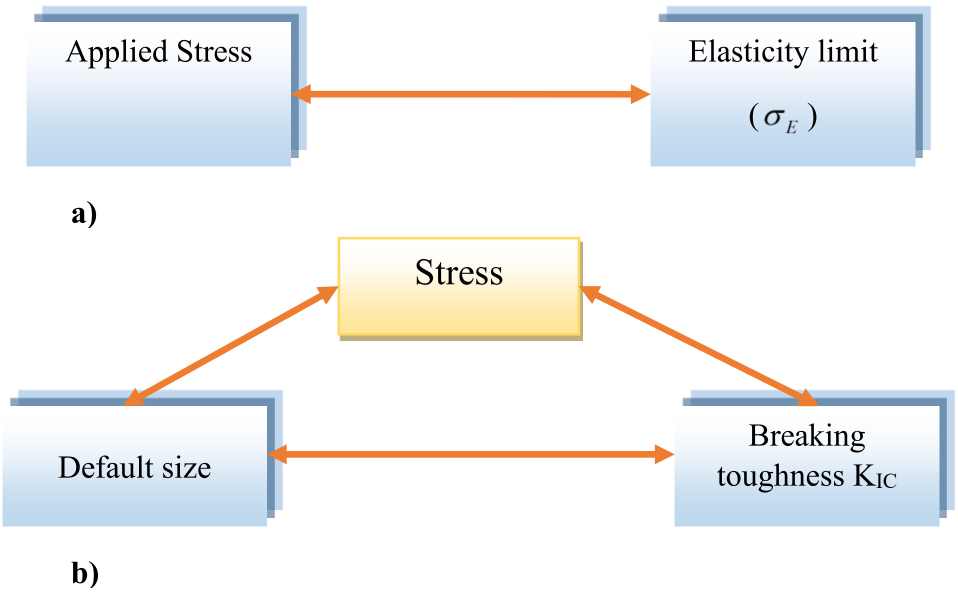


Figure III.12 : Comparison of the classic approach (a) and the approach using MLR (b).

III.2.1. Study of a cracked elastic medium

In a cracked elastic medium, the region close to the crack tip can be decomposed into three zones [86]. :

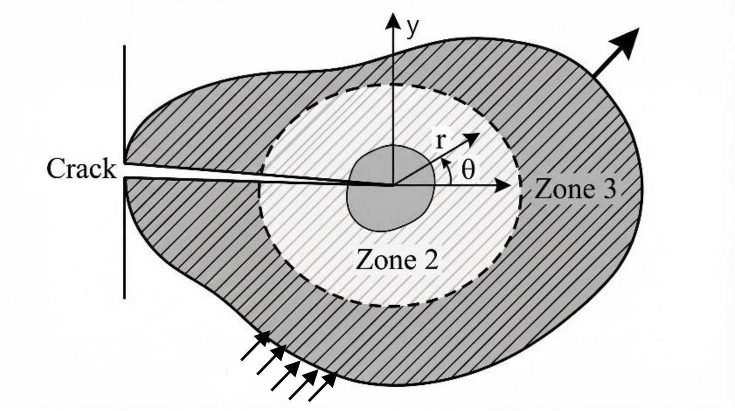


Figure III.13: Mechanical field zones

There are 3 **(Figure III.13)** categories of analysis depending on the size of the plastic zone at the crack head:

1) No plasticity:

Fragile breakup → linear fracture mechanics - calculation of K_I

2) Confined plasticity:

Determination of the extent of the plastic zone

Correction of the crack length and application of linear fracture mechanics.

The concentration of stresses at the crack tip creating local plastification, it is necessary that the size of this zone remains small in relation to the length of the crack, and the dimensions of the structure, so as not to disturb the elastic distribution too much. constraints.

3) Extended plasticity:

Nonlinear fracture mechanics (or plastic elasto) calculation of the J integral.

Extended plasticity is the domain for which the size of the plastic zone is no longer negligible compared to the length of the crack or the dimension of the ligament. It falls within the framework of fracture mechanics with non-linear behavior, and its study is only very recent.

The development zone defined in linear elasticity is replaced by a deformation field which only depends on the elastic limit in shear and the orientation relative to the axis of the crack (Prandtl field).

At the tip of the crack, the blunting and the Prandtl field lead to finite stresses.

III.2.2. Fracture modes

III.2.2.1. Crack and fracture modes

Fracture mechanics aims to describe the stages of initiation and propagation of cracking. Depending on the behavior of the material during the propagation of a crack, we can be confronted with two types of fracture:

- **Brittle fracture**:

In the absence of significant plastic deformation (linear fracture mechanics)

- **Ductile fracture**:

In the presence of non-negligible plastic deformation (non-linear fracture mechanics) (**Figure III.14**). In this case, depending on the extent of the plastic zone at the crack tip, we differentiate the case of confined plasticity from that of extended plasticity .

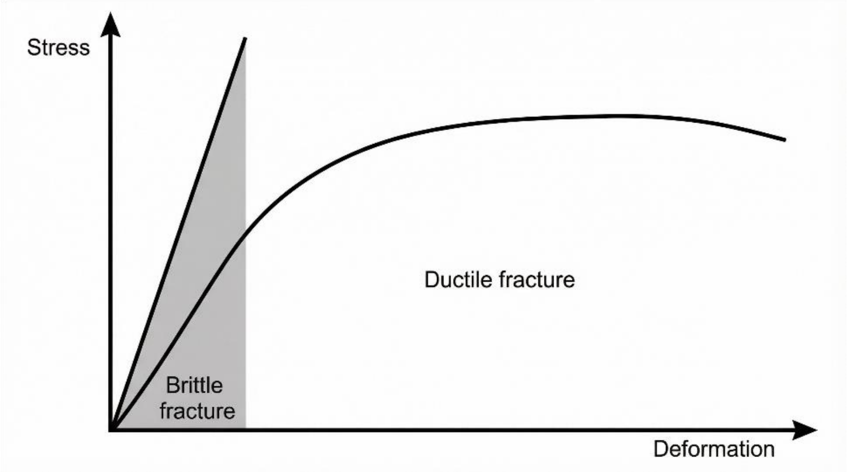


Figure III.14: Stress strain curve in the case of brittle and ductile fractures.

A plane crack is defined by the following parameters, its plane (n), its boundary (v) and the relative displacement of its faces (u) (**Figure III.15)**. The crack propagates in the direction normal to its boundary (v) according to three elementary modes of cracking, opening (I), plane shear (II) and anti-plane shear or tear (III).

The opening mode, or mode I, corresponds to the component of the relative displacement of the crack faces along the normal to the crack plane (n). The plane shear mode, or mode II, corresponds to the component of the relative displacement according to the direction of propagation (v). Finally, the anti-plane shear mode or mode III corresponds to the component tangent to the crack front **(Figure III.16).** In linear elasticity, the stress at the end of the crack is the linear superposition of the contributions of these three elementary modes.

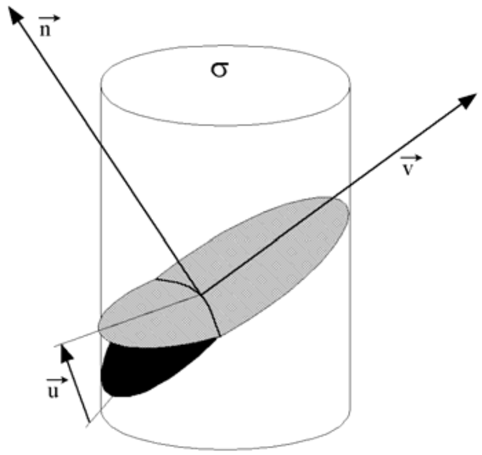


Figure III.15: Flat crack

Cracking manifests itself by the irreversible separation of a continuous medium into two parts, called crack lips, which introduces a discontinuity in the direction of movement. The possible movements of the lips of each crack are combinations of three independent modes [35]:

- **Mode I :** opening of crack lips.
- **Mode II :** plane shear.
- **Mode III :** anti-plane shear.

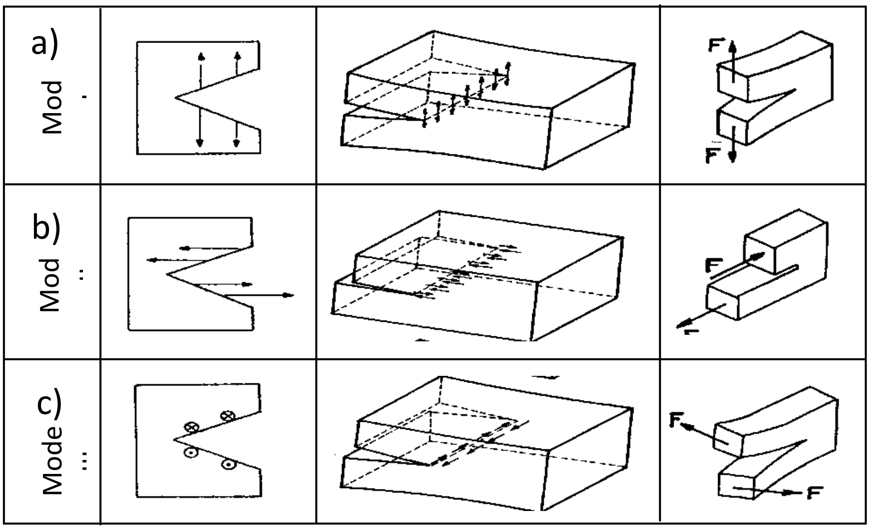


Figure III.16: Diagram showing the three modes of break: a) mode I; b) mode II; c) mode III

mode I is the most dangerous for the extension of a crack; however, once initiated and for mixed stresses or complex geometries, the crack tends to bifurcate, and therefore rarely remains straight (2D) or plane (3D).

For the three elementary cracking modes illustrated in (**Figure III.16**), these stresses were calculated by Irwin using elasticity theory. They are expressed by the mode relation I:

$$\text{Mode I} = \begin{cases} \sigma_{xx} = \dfrac{K_I}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\left(1 - \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \sigma_{yy} = \dfrac{K_I}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\left(1 + \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \tau_{xy} = \dfrac{K_I}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\sin\dfrac{\theta}{2}\cos\dfrac{3\theta}{2} \end{cases} \qquad \text{(III.1)}$$

III.2.2.2. Stress intensity factor

The analytical solutions for the three modes were determined similarly in linear elasticity. In plane stresses $\sigma_{zz}=0$ and $\kappa= (3-\nu)/(1+\nu)$ and in plane strains $\sigma_{zz}=\nu(\sigma_{xx}+\sigma_{yy})$ and $\kappa=3-4\nu$.

Thus the fields of stress, strain and displacement in linear elasticity are known, their shape does not depend on the length of the crack or the stress applied to the structure, only their intensity depends on it.

This intensity, also called stress intensity factor, varies linearly with the applied stress and evolves like the root of a characteristic dimension of the defect. We will note : $K_i = Y\sigma\sqrt{a}$ where **a** is the length of the crack, **σ** the applied stress and **Y** a geometric factor depending on the shape of the crack and the type of specimen.

Expressions of the stress intensity factor as a function of crack length and loading type for a large number of geometries and crack loading cases.

Finally, the relevant mechanical parameter for characterizing the behavior of a crack is therefore the stress intensity factor. [35]

III.2.2.3 Some statistics on rupture cases

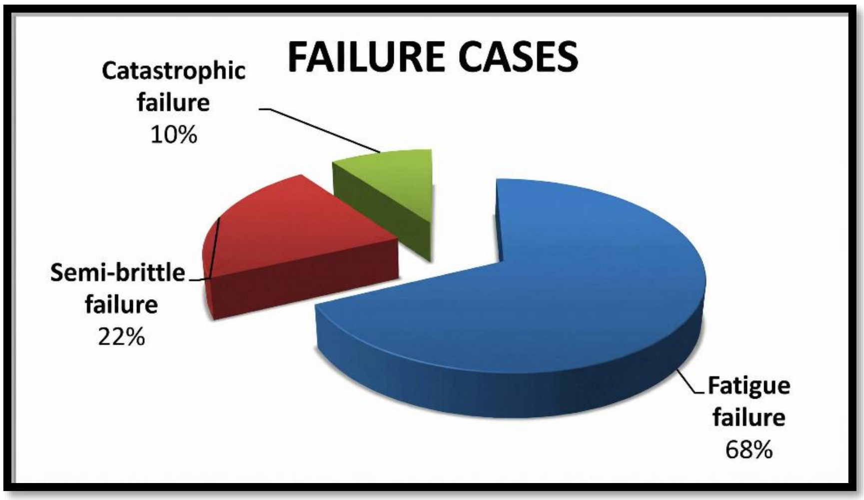


Figure III.17: The parameters in the destruction of a part

III.3. Fatigue damage mechanism

The following definition of fatigue: « *General term, used to designate the behavior of materials under repeated cycles of stress or deformation, which cause deterioration of the material, resulting in progressive failure* ».

Fatigue is the progressive deterioration of a structure by cracking which can lead to rupture, this structure being subjected to variable stresses nevertheless verifying the practical resistance criteria.

- **Fatigue limit :**

For a given average stress, the fatigue limit is the largest stress amplitude for which no failure is observed after an infinite number of cycles N [86].

- **Endurance :**

The fatigue resistance capacity of parts and assemblies [35].

- **Endurance Limit or lifetime :**

For a given average stress, the endurance limit is the largest stress amplitude for which 50% failure is observed after a finite number N of cycles [35].

- **Fatigue life or endurance :**

The number of stress cycles N necessary for the specimen or part to break for N stress cycles (σ_a , σ_m). These values can be determined using statistical methods.

*III.3.*1. Types of fatigue

Fatigue failure can be caused by several processes. Among these processes:

- **Fatigue-creep :** Damage caused by cyclic loads applied at high temperatures relative to the melting temperature of the material.
- **Thermal fatigue :** Temperature variations over time also generate thermal stresses responsible for the phenomenon of thermal fatigue.
- **Thermomechanical fatigue :** When the temperature and external mechanical stress vary over time.
- **Fatigue under corrosion :** when repeated stresses are in the presence of a corrosive environment.
- **Fretting-fatigue** : *results from stresses generated at interfaces by the relative movement of two solids.*

III.4. Crack initiating

Defining crack initiation remains one of the main difficulties. This can in fact be different depending on whether we are interested in the microstructural evolution of the material or whether we consider the appearance of a microcrack. This phase **(Figure III.20)** most often initiates on the surface of the part from persistent sliding bands or right next to inclusions, forming microcracks and propagating. These microcracks travel through a few grains following crystallographic directions

When a test piece or part gives rise to fatigue failure, its life can be broken down into three stages: I, II and III [36] **(Figure III.8).**

III.4.1. Crack damage stages

We are interested here in fatigue mechanisms, and in particular crack propagation by fatigue. The failure of a part or structure by fatigue can be divided into three stages [87]: initiation, the propagation of a main crack (or a few cracks) and the final sudden failure. Initiation occurs preferentially in areas where there is a concentration of stress: sudden change in section or geometry, inclusions or surface defects. However, initiation can occur in places that are perfectly polished and have no areas of stress concentration. For ductile crystalline materials, we see that, from a certain number of cycles, persistent sliding bands appear at the location of the deformations. Observation of these bands reveals the formation of extrusions and intrusions which constitute the beginnings of microcracks **(Figure III.18).** The initiation stage ends with the formation of a main crack, which propagates along the plane perpendicular to the axis of the main stress.

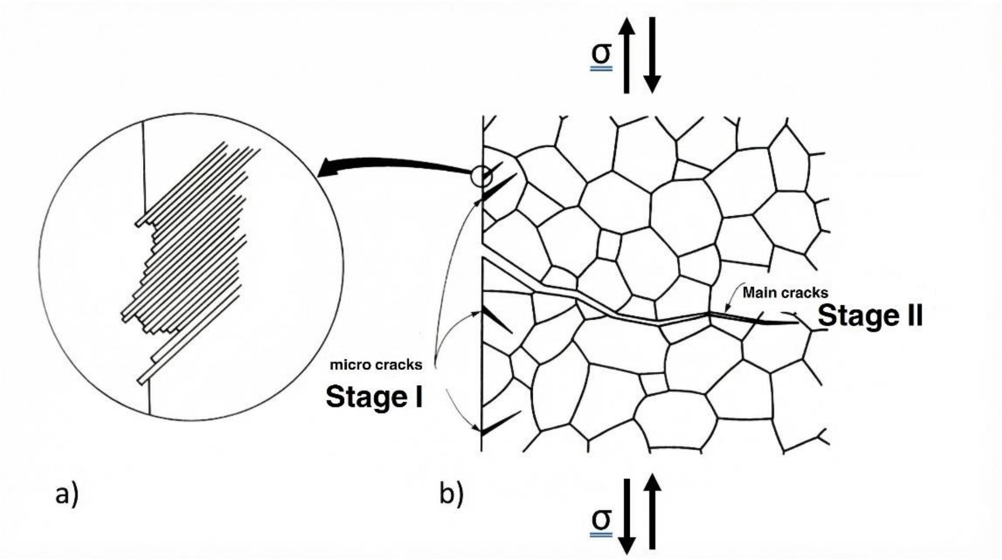


Figure III.18: Initiation of microcracks due to the formation of intrusions and extrusions [87, 88]

Microcrack initiation, **a**) alternating sliding and formation of intrusions and extrusions, **b**) main crack formation from microcracks [36].

Stage I represents the initiation and propagation phase of micro cracks, during this initiation phase, the defects progress in the sliding plane where they appeared. This plane of maximum splitting is most often oriented at 45° relative to the direction of the principal stress. This orientation most often takes place following a crystallographic direction that is very favorable to sliding.

By initiation, we mean the appearance of a crack of sufficient size to significantly modify the essential characteristics of a structure considered. The endurance limit therefore characterizes the maximum stress that the material can withstand before a crack begins [37].

The priming stages can be significant (up to 90% of the lifetime). Physically, the damage is far from being easily defined based on the number and size of cracks [38]. Thus, between the initiation of a micro crack and the propagation of a macroscopic crack, there exists a set of phenomena which, depending on the level of observation, can be assimilated to initiation or propagation [39]. The number of cycles at initiation therefore remains difficult to define, because it depends on the resolution of the means used [37].

Microscopic initiation generally concerns defects of the order of grain size, while macroscopic initiation extends up to defects of 1 mm. The domain of defects of size 1 to 100µm concerns microscopic cracks (micro cracks). Up to a size of the order of a millimeter, we are in the domain of short cracks beyond which we speak of long cracks to which the concepts of linear elastic fracture mechanics (MELR) apply. The combination of all these defects constitutes **"fatigue damage".**

III.4.2. Crack subjected to static loading

The static loading modes of structures include monotonic loading and cyclic loading. Each loading mode corresponds to a particular ruin mode. Only monotonic loading is taken into account in this study. For a monotonic loading, the loading increases continuously. Loading may consist of an imposed displacement or an imposed load. In the linear case (elasticity), the two cases are similar. For plastic behavior, there is a limit load that the structure cannot exceed. For force loading, a sudden rupture occurs due to plastic instability. This could be, for example, the case of an increase in pressure in a reservoir. [39]

The mechanisms of rupture involve the notions of constraint and energy. Rupture occurs, for the first notion, when the applied stress is greater than the rupture stress, and for the second notion, when the energy is equal to the rupture energy. Consequently, the cracking phenomenon is described by different approaches, through several parameters.

These different approaches constitute the basis of fracture mechanics. Therefore, in this paragraph, reminders of well-known theories will be presented. The interest of these reminders is to show the evolution of approaches and models. The first studies focused on cracks in elastic materials, then in plastic and viscoplastic materials. The approach to cracks subjected to static loading will constitute the basis of the approach to cracks subjected to dynamic loading.

III.4.3 Crack in elastic materials

Irwin's analysis of a static crack in a purely elastic regime makes it possible to note that the state of stress and deformation around a crack is known within one factor, which is KI, KII, or KIII (factors d stress intensity) depending on the crack opening mode.

These stress intensity factors are involved in the expression of the solutions as measures of singularities of the mechanical fields at the crack tip.

The stress field at the crack tip expressed at a point P as a function of the distance from the crack tip, r, and the angle relative to the crack plane, θ, **(Figure III.19)** expressed in the following form:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} \Sigma_{ij}(\theta) + 0 \qquad \text{(III.2)}$$

This solution is obtained by the method of Airy functions for hypotheses of plane stresses or deformations, in the case of a crack in simple opening mode (mode I). The KI factor depends on the applied loading and the geometry.

In this equation, Σij (θ) is a function of the angle measured relative to the crack plane (**Figure III.19)**.

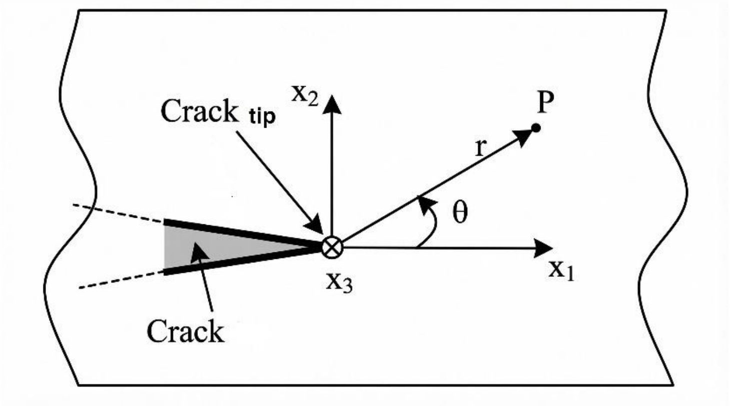


Figure III.19: System of axes in relation to the crack.

Only the stress intensity factor K characterizes the stress field. It depends on the geometry of the problem and the distant loading applied to the cracked structure. The stresses are infinite in r-1/2 at the crack tip and the stress intensity factors reflect the singularity at the crack tip. There is a relationship between the stress intensity factor and the stress concentration factor which allows the passage between the geometric defect and the crack [35]:

$$K = \lim_{r \to 0} \frac{1}{2} \sigma_{max} \sqrt{\pi r} \qquad \text{(III.3)}$$

a. Crack in materials with plastic phase :

In reality, plastic deformations appear in certain areas during rupture.

They induce non-linear behavior in the material. Plasticization occurs where stresses are highest, i.e. at the crack tip. As a result, this plastic zone modifies the stresses in the vicinity of the crack. The stress fields are then different from the elastic case. Non-linear rupture therefore relies on the study of this plastic zone making the approach more realistic, but also more difficult..

b. Perfectly plastic material (Irwin model) :

The Irwin model offers an estimate of the dimension of the plasticized zone (small dimension compared to the length of the crack) and of the new stress distribution. The determination of the plastic zone is done using the relations of the asymptotic solution of Westergaard and the Von Mises criterion. Irwin studied the crack in an infinite plate, stressed by a stress applied from afar. It obtained as value for the border of the plastic zone:

$$\mathrm{r_y} = \frac{1}{2\pi} \left(\frac{\mathrm{K_I}}{\sigma_\mathrm{e}} \right)^2 \qquad \text{(III.4)}$$

The stresses at the crack tip are limited in the plasticized region. The stresses in the plastic zone do not exceed the elastic limit, noted σe. This implies modifications in the distribution of stresses. The truncation, made in relation to the elastic case, must be compensated outside this zone, which results in an elongation (also equal to ry) of the plastic zone (**Figure III.20)**.

When the crack is accompanied by a plastic zone, it behaves like a fictitious elastic crack of length « a + ry », whose stress curve is similar to that in linear elasticity, but translated by a length ry. Plasticity has the effect of giving a larger opening surface than in the elastic case. In addition, it is possible to calculate the opening at the crack tip (called « Crack Opening Displacement » and denoted C.O.D.). It is determined at the crack tip (x = a), and is worth according to Irwin :

$$\delta = \frac{4}{\pi} \times \frac{K_I^2}{E_{\sigma_e}} \qquad \text{(III.5)}$$

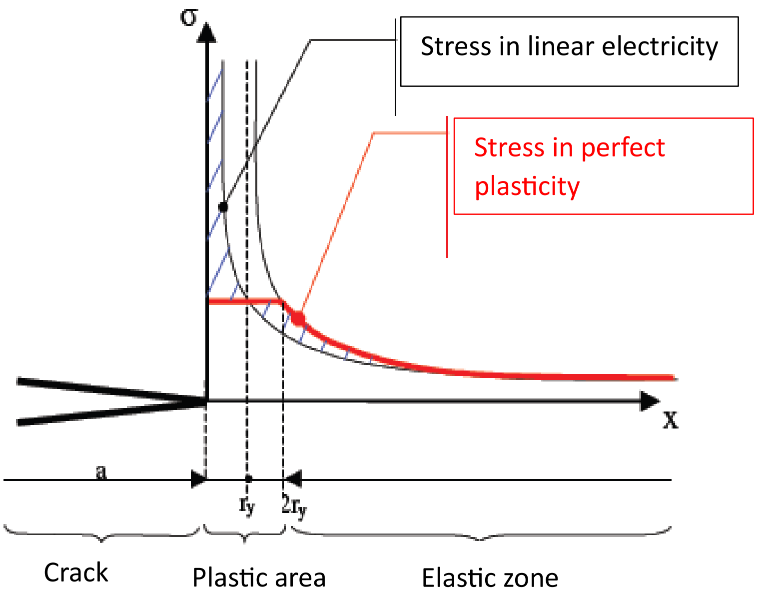
Just like Irwin's model, Dugdale Barenblatt's model is based on the solution obtained in linear elasticity for a crack, in the case of an infinite plate loaded in mode I. This model is therefore similar to that of Irwin (plasticized zone, plane constraints) but more elaborate because it takes into account the cohesion forces. Plasticity is modeled by a constant distribution of stresses equal to the elastic limit (**Figure III.21**).

The size of the plastic half-zone is :

$$r_y = \frac{\pi}{16} \frac{K_I^2}{\sigma_e^2} \qquad \text{(III.6)}$$

The crack opening is then equal to:

$$\delta = \sqrt{2} \frac{K_I^2}{E_{\sigma_e}} \qquad \text{(III.7)}$$



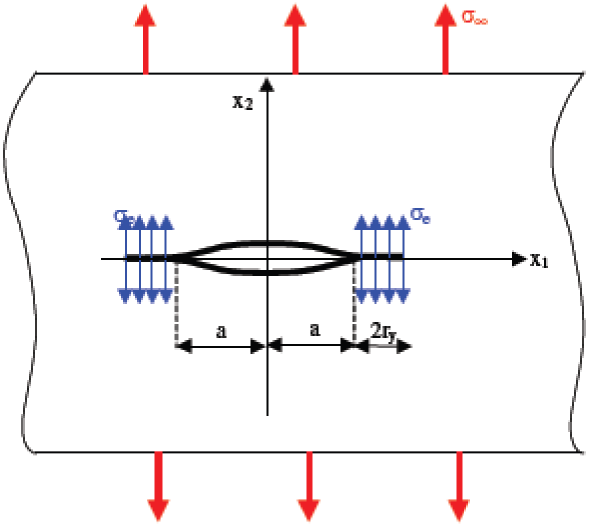


Figure III.20: Representation of Dugdale Barenblatt.

Figure III.21: Representation of the crack according to Irwin.

Measurement of K_{Ic} :

To measure K Ic it is therefore necessary to use test pieces for which the plastic zone is sufficiently small. The test specimen dimension criteria are of two orders:

the thickness must be sufficient to have a state of plane strain;

the length of ligament b of the specimen must be sufficient so that there is no possibility of relaxation of stresses by overall plastic deformation.

These considerations led ASTM to propose the following conditions for thickness

B and the initial length b of the ligament:

Initial length of the ligament........................ $b \geq 2{,}5\left(\frac{K_{Ic}}{R_p}\right)^2$

Thickness of the specimen........................... $B \geq 2{,}5\left(\frac{K_{Ic}}{R_p}\right)^2$

With :

R_P **:** elastic limit of the material considered.

The standardized dimensions of the test pieces are such that this last condition necessarily entails that:

If these conditions are respected, we obtain a valid measure of K_{Ic}

. This is highlighted in **(Figure III.21)** where the variation of G^c with the thickness of the specimen; We observe that G^c decreases and tends towards G_{Ic} when the thickness is sufficient.

II.4.4 Propagation of cracks by fatigue

The theory of fracture mechanics is an effective tool that allows designers and engineers to consider the development of defects in materials and so avoid any sudden crack. For static or dynamic loads, and based on the toughness of the material in question, the maximum loads that the structure containing defects can withstand can be calculated. Conversely, when the value of the load acting on the structure is known, one can deduce the critical size of tolerable defects.

However, it is only to consider the structures that surround us to see that they are subjected, in operation, to time-varying loads, whether it be the wings or landing gear of an airplane, or the components of any rotating machinery... Repeated application and removal of a load, even if its value is lower than the maximum allowable load calculated using the fracture mechanics approach, might result in structural failure. This concerns the phenomenon of tiredness.. [89].

The applied charge and, consequently, the constraint may fluctuate in a sinusoidal, periodic, or aperiodic manner as time progresses. However, in order to simplify the process, fatigue tests are conducted in laboratories using basic loads (sinusoidal or triangular). In this instance, the test's characteristic variables are (**Figure III.22**):

- The maximum stress$\sigma_{max}$and minimum σ_{min} ;
- stress amplitude σ_a =1/ 2(σ_{max}-σ_{min}) ;
- the average stress or static stress $\sigma_m = 1/\ 2(\sigma_{max} + \sigma_{min})$;
- the min max stress ratio R =($\sigma_{min}/\sigma_{max}$) ;
- The period T (or frequency f=1/T)

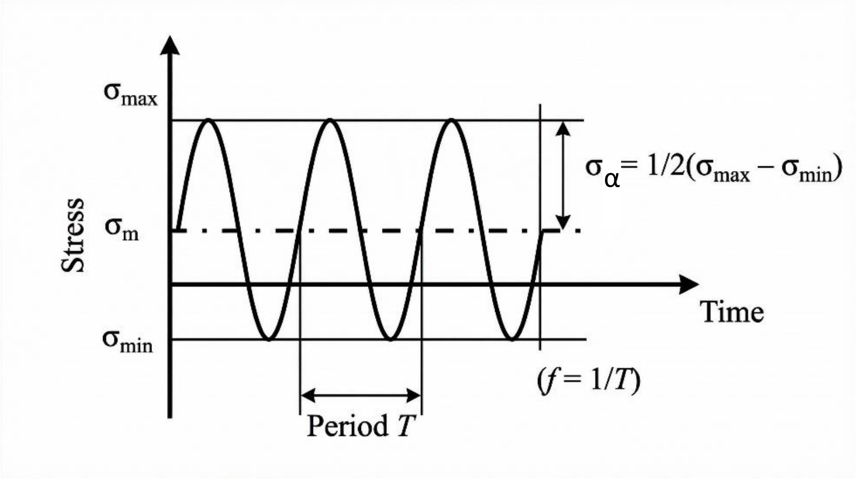


Figure III.22: Schematic representation of fatigue loading [88]

III.5. Fatigue influence parameters

Fatigue, whatever the mechanism, depends on numerous factors which may be of mechanical, metallurgical, geometric origin or linked to working conditions.

III.5.1. Metallurgical parameters

The production of a part always involves mechanical and thermal operations which allow its shaping and which act on the metallurgical state of the material used, therefore on its characteristics of use.

For this reason, we will examine the influence of various metallurgical factors on fatigue life, in order to present a mode of reasoning rather than an exhaustive list of these various parameters.

- **Grain size**

Fine-grained structures have better fatigue resistance than coarse-grained ones.

- **Orientation of fibering relative to the direction of forces**

The general orientation of the grains (fibering) gives the material a more or less marked anisotropy. The static characteristics and fatigue life will be better in the long direction of the fibering than in the other directions (long cross and short cross).

- **Work hardening rate**

The work hardening resulting from forming operations has the effect of consolidating the material (increase in the elastic limit), and consequently, improves fatigue resistance.

- **Thermal treatment**

Depending on whether the heat treatment causes a softening or hardening of the material, the fatigue life will be reduced or increased. Additionally, heat treatment can change grain size.

- **Metallurgical health of the alloy**

Metallurgical defects (vacancies, interstitial defects, precipitates, inclusions) can be the cause of fatigue damage. By incompatibility of deformations, they cause local stress concentrations. The drop in lifetime will depend on their quantities, size, nature, distribution and their orientations in relation to the efforts.

III.5.2. Mechanical and geometric parameters

- **Nature of loading**

The cyclic loading can be constant or variable amplitude (and even random, spectrum). In the case of constant amplitude loadings, the predominant parameters are:

- **The shape of the signal**:

A square type signal is more penalizing than a sinusoidal type signal.

- **The ratio R (ratio between the minimum value and the maximum value of the loading):**

At constant maximum stress, if R increases, the lifetime increases.

- **The average stress**:

At constant loading amplitude, if the average stress increases, the lifetime decreases.
The period of the signal generally has little influence on the lifetime for metals. This rule is invalidated when the fatigue phenomenon is associated with other modes of damage.

- **Depending on time:**

Fatigue-corrosion, fatigue-creep, or when the rapidity of the stresses produces heating.

- **In the case of variable loadings, the predominant parameters are:**

The presence of overloads: the periodic repetition of an overload can delay the propagation of cracks.

the order of appearance of the cycles.

- **Shape accidents (discontinuity in the geometry notches, holes, etc.):**

A formal accident locally increases the level of constraint. This increase can be translated by an elastic stress coefficient Kt: ratio between the maximum local stress and the nominal stress. In the limited endurance domain (domain targeted by the aeronautical industry), if the value of Kt increases, the lifetime decreases.Any change in section results in a reduction in lifetime for an amplitude ,Cyclic stress data.

- **Notch effect:**

It is an action analogous to a superficial weakening exerted by any accident of shape constituting a notch not parallel to the main stress responsible for the cracking. Two cases must be considered: notch effect perpendicular to the main stress and oblique notch effect. For a circular section this element can, if the notch effect is significant enough, reverse the curvature of the front, which becomes concave. This makes it possible to assess the overall notch effect responsible for progressive cracking. If there is an obliqueness of the notch effect with respect to the general direction of the stresses, the crack first follows the bottom of the notch, the crack is subdivided into a series of distinct cracks, orthogonal to the direction general constraints, and more or less intertwined with each other. It is mainly on parts working in torsion that we observe breaks of this type.

- **Scale effect :**

At an equal stress level, two parts of the same geometry but different dimensions will not have the same fatigue resistance: the more the dimensions of a part increase, the more its fatigue resistance decreases. This observation is mainly explained by the volume of material used: the larger this is, the greater the probability of having metallurgical defects. We can cite three main causes of the scale effect:

- A mechanical cause.
- A statistical cause.
- A technological cause.

Machining quality :

Generally, fatigue damage first appears on the surface of parts. Taking the following two aspects into account improves fatigue resistance:

the micro-geometry aspect of the surface: poor machining causes micro-reliefs on the surface likely to locally increase the stress level; the initiation of surface cracks is therefore delayed when the roughness is low;

the “residual stresses” aspect: machining can introduce residual tensile stresses on the surface (they are balanced in depth by residual compressive stresses). These stresses, superimposed on those of the loading, accelerate fatigue damage.

- **Effect of surface condition**

Surface roughness is always accompanied by a decrease in fatigue strength. This effect is comparable to that caused by micro-notches.

- **Effect of environment**

An aggressive environment (high temperatures, corrosive environments, etc.) aggravates the phenomenon of fatigue. New phenomena appear such as creep or corrosion. Their action is linked to the exposure time.

It results from the experiments that the reduction in fatigue resistance by corrosion, before the test is generally small, while the influence of the corrosion exerted during the fatigue test is considerable [33].

The influence of corrosion can therefore be considered from two points of view, depending on whether:

- Fatigue forces are exerted on previously corroded parts.
- Fatigue and corrosion act simultaneously. This process is called corrosion fatigue..

III.6. Characterization of crack propagation

Fatigue mechanisms show that it is often necessary to take into account not only the possibility of crack initiation, but also evaluate their possible propagation, in order to ensure that the cracks do not reach the critical length which will inevitably lead to a sudden rupture of the piece. For this, it is necessary to have quantitative data characterizing the speed of propagation of a fatigue crack.

In the 1960s, Paris et al. [90, 91] showed that there is a relationship between the cracking speed (da/dN) and the stress intensity factor K. Since the stress value varies, we define the amplitude of the stress intensity factor stress intensity: $\Delta K = K_{max} - K_{min}$ where K_{max} and K_{min} are the extreme values of K during the cycle. On a bilogarithmic scale, the propagation speed (da/dN) as a function of ΔK has the appearance given in **(Figure III.23),** on which we distinguish three regimes.

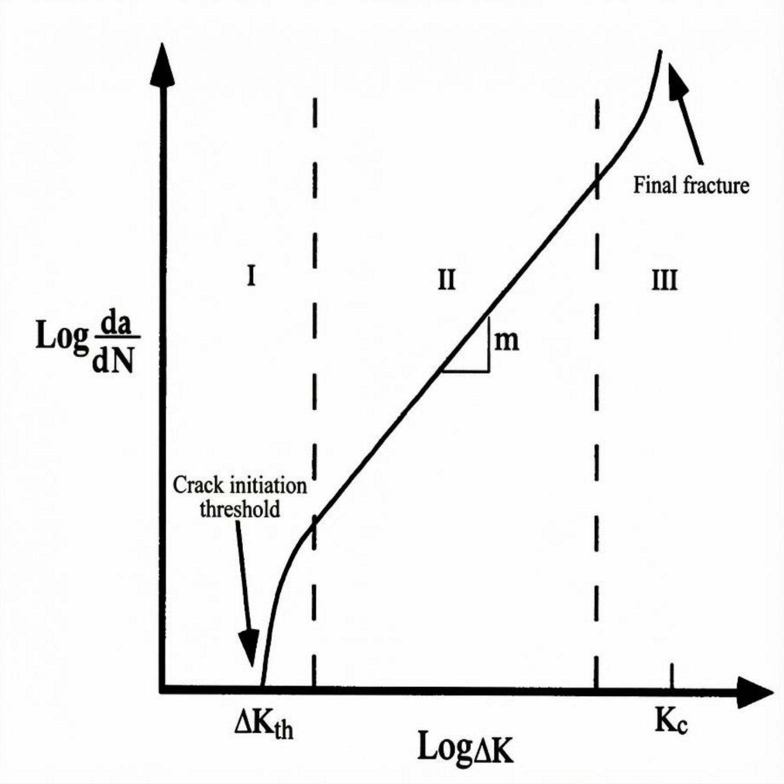


Figure III.23: Fatigue crack propagation regimes

Regime I is characteristic of the propagation threshold. The propagation speed tends towards zero for a value ΔK_{seuil} called propagation threshold.

Regime II, called the Paris regime, is entirely within the scope of our work. This regime is characterized by the progressive acceleration of the propagation speed when ΔK increases. Paris et al. [91] linked the propagation speed and the amplitude of the stress intensity factor ΔK by a power-type relationship, called Paris laws.:

$$\frac{da}{dN} = C(\Delta K)^m \qquad \text{(III.8)}$$

Where C and m are experimental parameters depending on the material and the test conditions (load ratio, environment, etc.).

Regime III corresponds to a very rapid acceleration of the cracking speed. The stress intensity factor is close to the critical value C K corresponding to final failure.

However, Paris law is not universally applicable. The work of Elber [92] showed that the role of closure must be taken into account to describe the behavior of a fatigue crack. This phenomenon consists of the crack lips coming back into contact when the cyclic load becomes low. The part of the loading cycle during which the crack is closed is considered ineffective for the propagation mechanism **(Figure III.24).**

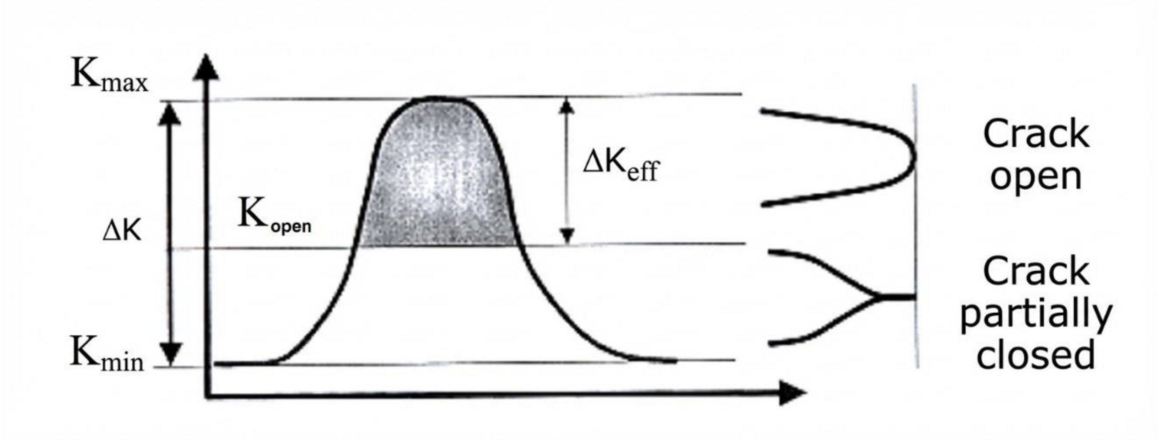


Figure III.24: Crack closure phenomenon

The closure effect is attributed to three distinct effects:

- Residual deformation in the plastic wake of the crack
- The roughness of the fracture facies
- Oxidation of fracture surfaces

Elber therefore proposed to define an effective value of the loading with an amplitude

effective stress intensity factor ΔKeff = Kmax – Kouv. A correction of the Paris equation is then introduced

$$\frac{da}{dN} = C(\Delta K_{eff})^m \qquad \text{(III.9)}$$

III.7. Thermal fatigue phenomenon

Under certain conditions of use, a material may be exposed to more or less sudden variations in temperature. These temperature variations cause dimensional variations in the material. Therefore, when the material is not free to expand or contract, it is subject to thermal stress.

Even in a non-recessed room, thermal stresses appear in the presence of a temperature gradient. When the temperature gradient is not very high, and its value is stable, thermal constraints can be reduced. These thermal constraints depend on the characteristics of the temperature gradient and the thermal properties of the material. If thermal stresses vary cyclically, we then speak of thermal fatigue, a phenomenon that can even affect metals.

The formation and development of cracks or micro-cracks due to thermal fatigue can lead to the complete degradation of the material or part. Material damage by thermal fatigue can be encountered on several industrial components: turbine blade [93, 94], train brake disc [95,96] or component of the nuclear reactor cooling circuit [97,98]…

III.8. Studied tools (Extrusion Die)

A die is a very hard mechanical part (**figure III.25**) used to shape a material (steel, aluminum, copper, plastic) generally by compression. The material is pushed hot or cold inside a shape which forces the material to take the desired profile. We can thus manufacture long objects, of constant section, either solid (bare or coated wires1, cords, strips, etc.) or hollow (pipes, profiles, etc.);

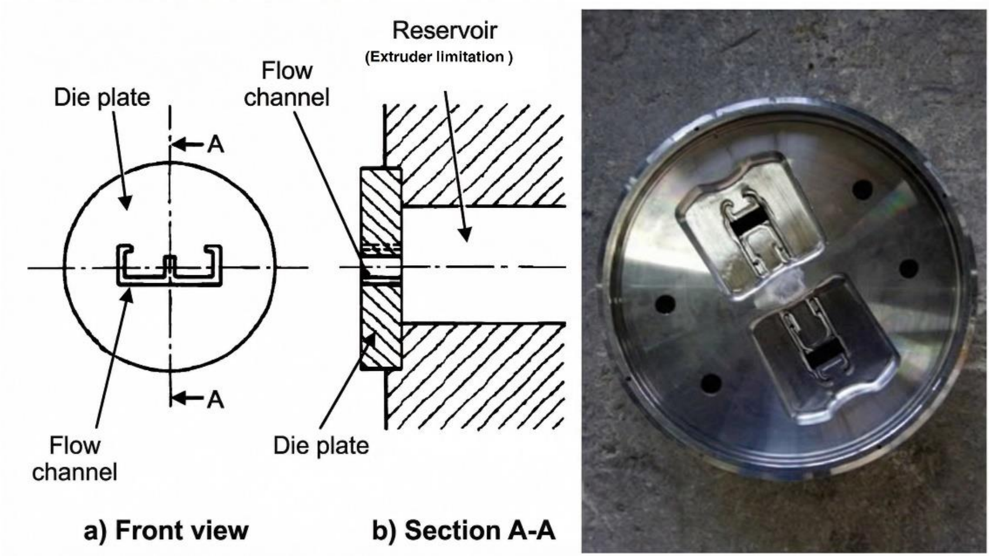


Figure III.25: profile dies [99].

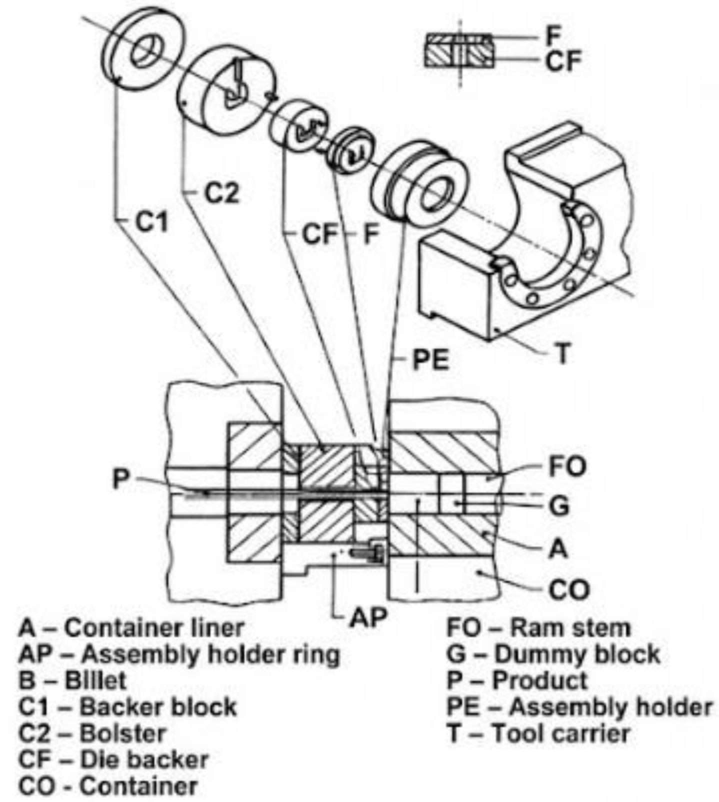


Figure III.26: Typical assembly of dies for solid profiled products [61].

1. H13 steel dies and Scope of use

It is a hot work steel with very good mechanical resistance at high temperatures, good toughness and good resistance to the formation of hot cracks. It can be cooled in water.

It is suitable for tools for hot work subject to high forces, mainly: needles, dies and containers in hot spinning presses for tubes and profiles in materials, tools for the manufacture of hollow parts, screws and nuts, rivets and bolts; die casting tools, press stamps for serial articles, application of dies, hot shear blades, molds for plastics.

1.1. Chemical composition

Table III.3: chemical composition of H13 steel [100].

Element	C	Si	Cr	Mo	V	Mn	P max	S max
Percentage %	0.35 to 0.42	0.80 to 1.20	4.80 to 5.50	1.20 to 1.50	0.85 to 1.15	0.25 to 0.50	0.030	0.020

1.2. Nomenclature :

Table III.4: nomenclature of H13 steel according to different standards [35].

Standard	AFNOR	AISI	UNE	Werkstoff-Nr	DIN
Nomenclature	Z 40 CDSV 5-1	H13	F 5318	1.2344	X40CrMoV5-1

1.3. Heat traitements

The die before use receives the following heat treatments:

Annealing at a temperature of 750°C up to 800°C, cooling is slow in the oven, and the maximum brinell hardness after annealing can reach 235 HB.

Stress relief annealing at a temperature of 600°C, cooling is slow in the oven. This treatment is used for relaxation after major machining or for complicated shaped tools. The temperature maintenance time after heating is 1 to 2 hours in a neutral environment.

Quenching at a temperature of 1020°C up to 1080°C, cooling is done in oil, in a salt bath (500-550°C) or in air and the duration of maintenance at the temperature of quenching is calculated in minutes by the following formula:

$$\mathbf{20+\frac{thiknes\ (mm)}{2}}$$

The aim of this treatment is to achieve a hardness of 52 to 56 HRC by air cooling.

- An income that is made in three stages; a first temper to the maximum of the secondary hardness, then a second temper to reach the hardness of use, and finally a third temper to eliminate tensions, at a temperature of 30°C up to 50°C below maximum tempering temperature.

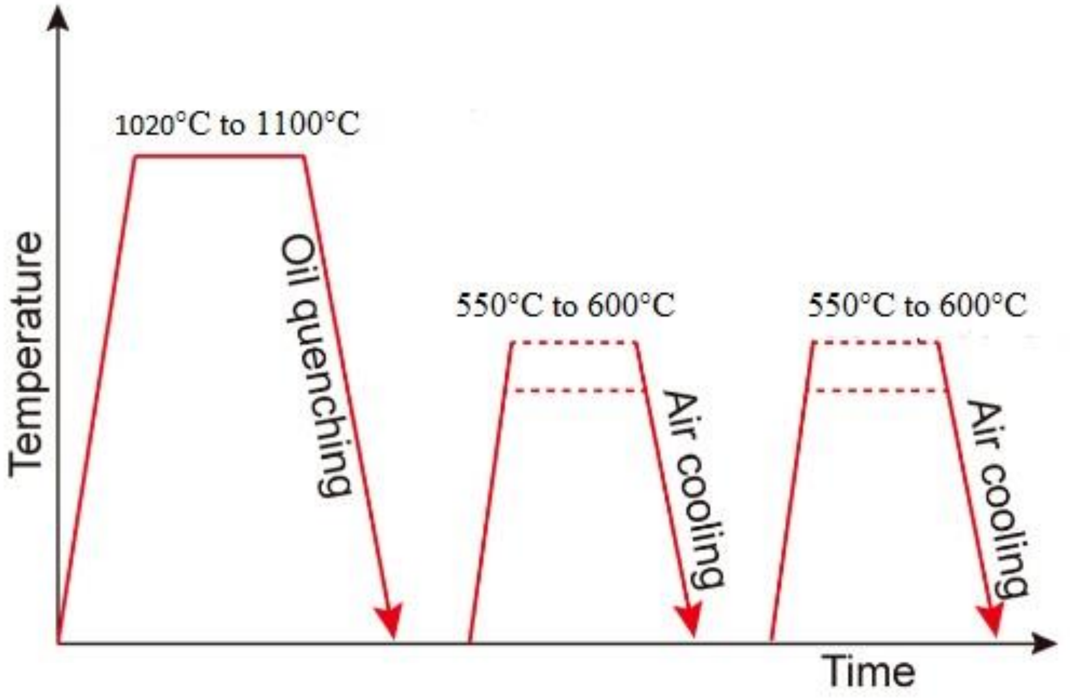


Figure III.27: heat treatments of H13 dies

1.4. Physical and mechanical properties

Table III.5: mechanical and physical properties of H13 steel [100].

<table>
<tr><td>Hardness (annealed) max</td><td colspan="2">Quenching temperature °C (±10°C)</td><td>Quenching medium</td><td>Tempering temperature (°C)</td><td colspan="2">Hardness HRC min</td></tr>
<tr><td>229</td><td colspan="2">1020</td><td>Oil</td><td>550°C</td><td colspan="2">50</td></tr>
<tr><td colspan="7">Coefficient of Thermal Expansion (by $10^{-6}.K^{-1}$)</td></tr>
<tr><td colspan="2">20 to 200 °C</td><td colspan="3">20 to 400 °C</td><td colspan="2">20 to 600 °C</td></tr>
<tr><td colspan="2">11.3</td><td colspan="3">12.4</td><td colspan="2">13.1</td></tr>
</table>

III.9. Defects in extrusion Die

Die failure encompasses various wear factors, predominantly influenced by the material quality and temperatures experienced during extrusion processes. The die is responsible for forming the shape of the extrusion, and it is kept in place by the die holder/ring so that it does not collapse or fracture. The die backer is responsible for providing support for the die. The extrusion load is transferred from the die to the pressure ring, transferring the force to the press platen. This process prevents the bolster from deflecting and ensures the weight is correctly transferred. The complete die set, which includes the die ring and the bolster, is being held in the press by the die slide, it also withstands a high degree of stress based on AISI H13 material used, and its high properties to withstand stress and fatigue. [101,102], as illustrated in **(Figure III.28).**

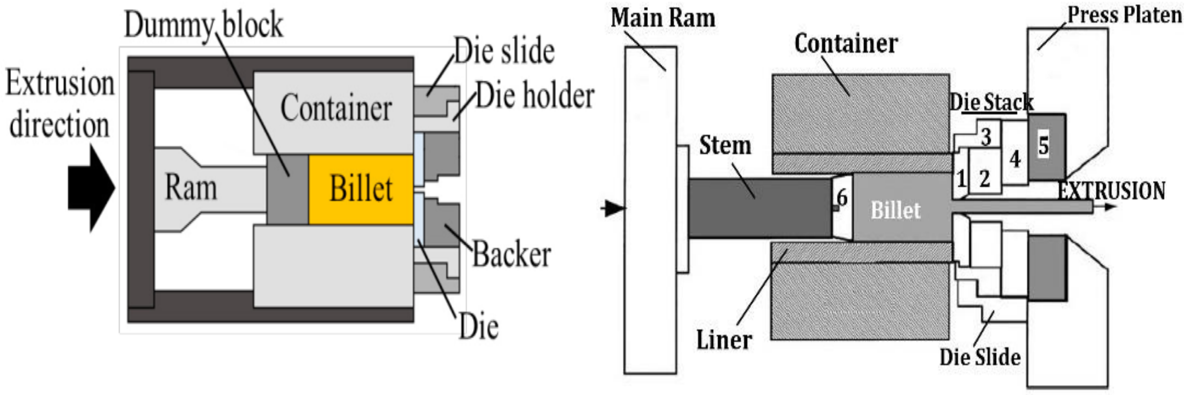


Figure III.28: Extrusion tooling used in hot extrusion. [102, 10],1, solid die; 2, backer; 3, die holder ; 4, bolster; 5, pressure pad; and 6, dummy block.

Factors contributing to die wear involve abrupt temperature fluctuations and prolonged exposure to high temperatures, resulting in friction between the aluminum flow and the die's surface.

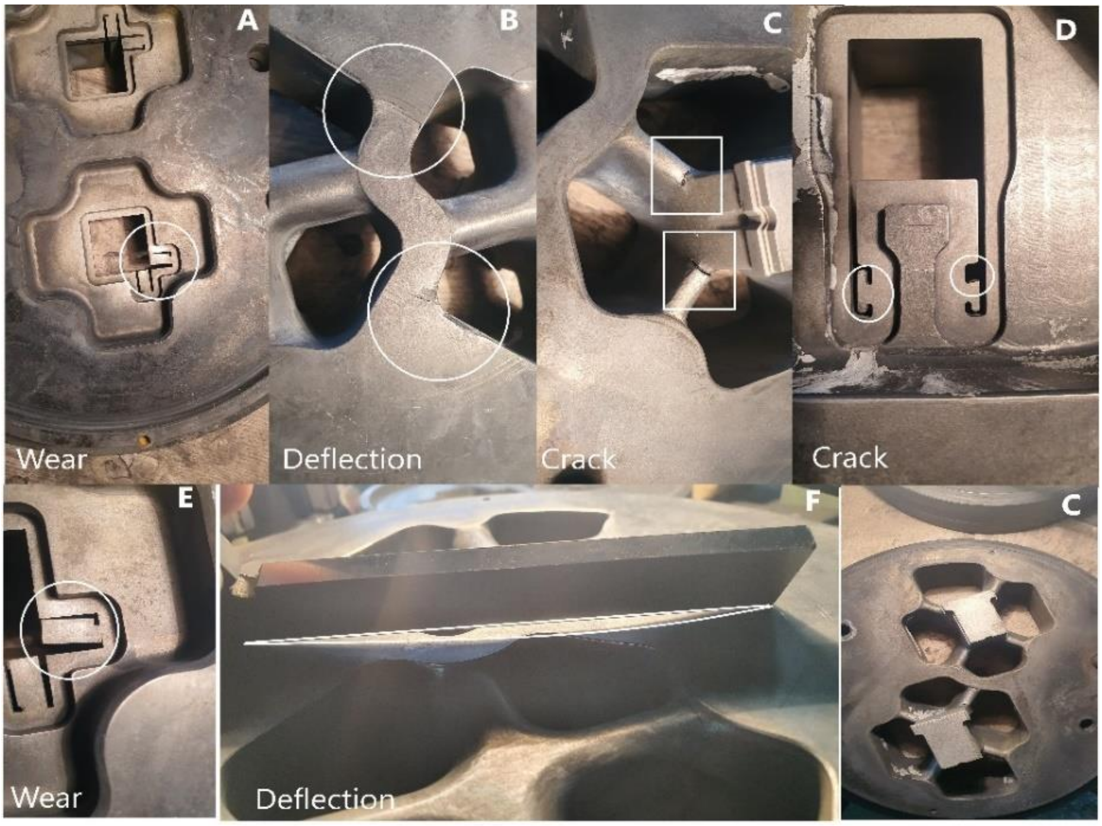


Figure III.29: The most important failures of aluminum extrusion dies

Among the factors mentioned, temperature and extrusion speed are of paramount importance. The speed of extrusion and the temperature increase at the die bearing are directly interrelated [102]. The extrusion die is susceptible to various categories of defects, with the most influential ones being those that are irreparable, including cracking, deflection, friction, and wear. (**Figure III.29**) illustrates some of these defects: (C, D) depict cracking in its advanced stages, while (B, F) illustrate deflection, and (A, E) showcase wear. Each of these defects contributes to a damage model in hot extrusion. A statistical examination of the primary types of die failure and their subcategories, as well as a greater understanding of die failure modes and processes, was conducted by AFM ARIF [103]. Indeed, the presence of various irreversible effects like wear, deflection, and cracks in a damaged die underscores the need to establish a reliability index and conduct a statistical analysis. This analytical approach is essential for pinpointing the pivotal factors that impact the lifetime of the die.

III.10. Conclusion

This chapter provided a comprehensive overview of the principles of fracture mechanics and their application to damage and failure analysis. Understanding fracture behavior is crucial for predicting crack initiation and propagation, which are among the most important degradation mechanisms affecting these components under extreme loading conditions. This chapter also discusses extrusion dies and work tools affected by many fracture mechanics factors during their operation, in particular High temperature.

Thermal fatigue is exacerbated by the presence of temperature gradients and thermal stresses within the die, as well as the potential for oxidation and hot corrosion at elevated temperatures. These factors can contribute to the formation of surface and subsurface cracks, ultimately leading to premature die failure.

CHAPTER IV

MECHANICAL-RELIABILITY STUDY

Chapter IV

IV .1. Introduction

The main aim of this chapter is to calculate the reliability index by taking into account the die material parts and the factors that affect them, based on the aforementioned considerations and recent research on extrusion die behavior and defects. In order to accomplish this objective, we have created a model that examines the behavior of the extrusion process by incorporating mechanical dependability and crucial affecting elements. As a result, a mechanical model has been created that integrates the rheological model and the damage model.

The current study focuses on developing a model to analyze the behavior of the crucial tool (die) in the aluminum extrusion process. This model combines mechanical reliability to determine several important parameters. A mechanical model is developed that combines the rheological model with the damage model. This model is used to assess the reliability index β and analyze the impact of random fluctuations in the input parameters of the aluminum extrusion process. The probability of die failure can be estimated using the integrated model, which is described in relation to the reliability index β determined by the reliability simulations in PHIMECA Soft® as a function that defines the limit state.

The reliability and the possible lifetime of the aluminum extrusion die can be affected by the variety of the extrusion conditions such as the applied extrusion load, the temperature of the extrusion billet and the die, the ingoing material quality, the velocity of the extrusion process, and the kind and form of the applied lubrication. In the extrusion process, it has been reported that the problems related to the die will be caused mainly by thermal fatigue, which is the result of temperature fluctuation and thermal stress on the die surface. With time, the development of the die temperature and temperature fluctuation, the increase of the number of stresses and thermal fatigue cycles, the die cracks and strains will be increased, which could then result in the prospect of die failure events. The relationship between the temperature/load, the number of fatigue cycles, and their possible failure lifetime was of interest and many studies were reported.

IV.2. Model Identification

IV.2.1. Basic Equations

As a general rule, it is important to note that cracks are often found in dies and extruders as a result of metal alloy die manufacturing processes. These cracks are usually about 0.01 mm deep during processes such as surface hardening of the bearing zone [104].

The increase in crack intensity, influenced by various material properties, results in heightened stress levels. This, in turn, accelerates crack growth at a certain rate. When the material is subjected to a specific stress intensity factor K for a given number of extrusions ΔN, the fracture length is accelerated and experiences a notable increase represented by the parameter 'a.' The rate of crack growth acceleration is determined by the changing property of interest within a strain resistance range, [105] expressed as the ratio Δa/ΔN. Consequently, we present the rate of acceleration of crack length, denoted as da/dN.

$$\frac{da}{DN} = C(\Delta K)^m \qquad \text{(IV.1)}$$

Constants unique to the material and environment under consideration are the factors C and m. In the case of the extrusion die steel.

IV.2.2. Lifetime determination

By substituting the definitions of maximum and minimum stress into the stress range formula, we can derive a valuable alternative definition of the stress range:

$$\Delta K = \alpha \Delta\sigma \sqrt{\pi a} \qquad \text{(IV.2)}$$

Here, α represents the crack geometry factor. In general cases, the rate of propagation of a given crack under constant amplitude loading depends on several factors, including the duration of the extrusion cycle, the end stresses of the fatigue cycle (σ_max, σ_min), and the length of the crack. In simpler scenarios where the similarity condition is met, the concept of stress intensity factor allows us to consider the two principal components by utilizing the range of stress intensity factor. This is because the extrusion cycle commences with the minimum load (Δσ = σ_max - σ_min) we consider that σ_min=0 in this case Δσ = σ_max.

By substituting Equation IV.2 into Equation IV.1 and integrating it, we can determine the total number of extrusions allowed before reaching the end of the fatigue life. The fatigue life (Nf) was determined by the number of cycles until fracture failure [106].

$$Nf = \frac{(a_0)^{1-\frac{m}{2}} - (a_c)^{1-m/2}}{C(\frac{m}{2} - 1)\alpha^m \pi^{m/2} {\sigma_{max}}^m} \qquad \text{(IV.3)}$$

Considering that H13 tool steel belongs to the category of ultrahigh strength steel, it is possible to determine the values of C and m for this type of steel by referring to standard sources. As previously mentioned, heat-treated and surface-hardened H13 steel has preexisting cracks (a0) that fall within a range of 0.05 to 0.1 mm. It is possible to set the geometry factor (alpha) value at 1.12 if the kind of fracture is an edge crack [104].

The equation alpha = f(a/w) is a function of the normalized crack length (a/w) and serves to eliminate surface tensile stresses. Using mode-I stress intensity factor according to its typical definition, while disregarding the finite-size factor f(a/W), and recognizing that the crack becomes unstable (a=ac) when KI = KIC, we can derive the following relationship [34].

$$a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{\alpha\sigma_{max}}\right)^2 \qquad \text{(IV.4)}$$

The behavior of the production system is governed by a mathematical law that describes the required deformations needed to shape a billet into a profile with the desired dimensions and shape. In the context of extrusion, the most suitable choice is the rheological law developed by Hansel & Spittel. The Hansel-Spittel model was selected due to its strong support for establishing a simple connection between variables such as strain, strain rate, and temperature. This enables the modelling of flow stress by considering the strain's dependence in accordance with the rheological law parameters [107,108]. It has been integrated into many alloys [109,110]. The modeling of behavior characterized by Hansel & Spittel's law has been seamlessly integrated into FORG shaping calculation codes.

$$\sigma_{max} = Ae^{m_1 T} T^{m_9} \varepsilon^{m_2} e^{m_4/\varepsilon} (1+\varepsilon)^{m_5 T} e^{m_7 \varepsilon} \dot{\varepsilon}^{m_3} \dot{\varepsilon}^{m_8 T} \qquad \text{(IV.5)}$$

Most referenced material has null values for parameters m5 to m9. Thus, it is possible therefore simplify the equation like this (equation IV.6).

$$\sigma_{max} = Ae^{m_1 T} \varepsilon^{m_2} e^{m_4/\varepsilon} \dot{\varepsilon}^{m_3} \qquad \text{(IV.6)}$$

In the given context, A represents the material's cohesion, T denotes the die temperature, and m1 signifies the material's sensitivity to temperature. Additionally, m2 and m4 dictate the material's sensitivity to tension, while m3 is contingent upon the material's sensitivity to the strain rate. Equation (IV.3) is employed to substitute the term a_c.

$$Nf = \frac{(a_0)^{1-\frac{m}{2}} - \left(\frac{1}{\pi}\left(\frac{K_{IC}}{\alpha(Ae^{m_1 T}\varepsilon^{m_2}e^{m_4}/\varepsilon\dot{\varepsilon}^{m_3})}\right)^2\right)^{1-m/2}}{C(\frac{m}{2}-1)\alpha^m\pi^{m/2}{\sigma_{max}}^m} \quad \text{(IV.7)}$$

The reliable geometric mechanical model is derived from the following equation (Equation IV.8). By combining the material model based on Hansel-Spittel with the mechanical model for lifetime analysis, we have developed a comprehensive mechanistic model for conducting reliability analyses.

$$Nf = \frac{(a_0)^{1-\frac{m}{2}} - \left(\frac{1}{\pi}\left(\frac{K_{IC}}{\alpha(Ae^{m_1 T}\varepsilon^{m_2}e^{m_4}/\varepsilon\dot{\varepsilon}^{m_3})}\right)^2\right)^{1-m/2}}{C(\frac{m}{2}-1)\alpha^m\pi^{m/2}(Ae^{m_1 T}\varepsilon^{m_2}e^{m_4}/\varepsilon\dot{\varepsilon}^{m_3})^m} \quad \text{(IV.8)}$$

Lemaître and Chaboche are modelled the damage expressed as a function of n/Nf is: (Equation IV.9) [108, 111].

$$D = 1 - \left[1 - \left(\frac{n}{Nf}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\beta+1}} \quad \text{(IV.9)}$$

Where α is given by:

$$\alpha(\sigma_{max}, \bar{\sigma}) = 1 - a\left(\frac{\sigma_{max} - \sigma_I(\bar{\sigma})}{\sigma_u - \sigma_{max}}\right) \quad \text{(IV.10)}$$

Equation IV.11, which describes a model correlating the Hansel–Spittel model, is thus the method by which the reliability engineering mechanical model is obtained [110]. (Equation IV.6) presents the mechanical model for reliability evaluations by combining it with the Lemaître–Chaboche model (equation IV.9) [108, 111].

$$D = 1 - \left[1 - \left(n \Big/ \frac{(a_0)^{1-\frac{m}{2}} - \left(\frac{1}{\pi}\left(\frac{K_{IC}}{\alpha(Ae^{m_1 T}\varepsilon^{m_2}e^{m_4}/\varepsilon\dot{\varepsilon}^{m_3})}\right)^2\right)^{1-m/2}}{C(\frac{m}{2}-1)\alpha^m\pi^{m/2}(Ae^{m_1 T}\varepsilon^{m_2}e^{m_4}/\varepsilon\dot{\varepsilon}^{m_3})^m}\right)^{1 \big/ 1-\left(1-a\left(\frac{\sigma_{max}-\sigma_I(\bar{\sigma})}{\sigma_u-\sigma_{max}}\right)\right)}\right]^{1/\beta+1} \quad \text{(IV.11)}$$

And then we construct failure steps from (equation IV.11) using the reliability statement with limit state functions to achieve the reliability index of the extrusion dies.

IV.3. Die Reliability Assessment

When delving into the realm of reliability within mechanics, it is essential to first establish the context in which the proposed approach operates. Reliability encompasses both well-established methods, such as the implementation of statistical techniques for manufacturing control, and emerging methods that focus on failure and risk assessment. The latter approach stems from a relatively new philosophy that must be contextualized.

Our interest lies in methods primarily developed in the context of material and structural modeling. These methods allow mechanics to draw upon the wealth of knowledge derived from fields like probability theory and the experiences gained in the domain of aluminum extrusion die manufacturing.

Embracing a probabilistic approach, reliability methods in mechanics enable us to calculate the reliability index and sensitivity to failure. The success of the dimensioning process is validated by confirming an equality function, which relates the number of extruded billets (determining die lifetime, Nf) to critical damage (Dc) and the acting damage (D) as time-independent output variables. This approach allows us to describe the structural state through a single global random variable known as the margin, representing the elementary resistance-solicitation case.

The limit state function, denoted as G(Xi), quantifies the lifetime limit, presented as the variation between the number of billets extruded into the die and the required number for satisfactory performance (Equation 12).

$$G(X_i) = D_c - D(X_i) \qquad \text{(IV.12)}$$

according to the literature [112,113,114,115], Dc is defined by: $0.2 \leq D_c \leq 0.5$

The reliability index β is employed to describe the probability of failure, which is defined as the shortest distance in the middle of the origin and the domain of failure in the equivalent Gaussian space u_i.

$$\beta = minimise\sqrt{\sum_i u_i^2} \text{ subjected to} G(X_i) \leq 0 \qquad \text{(IV.13)}$$

Ensuring the reliability of structures is a fundamental criterion when it comes to making choices in terms of design and maintenance. For each dimensioning rule, a failure scenario is described by means of a performance function $G(X_i) = R(X_i) - S(X_i)$

(X_i)being the basic random variables,R(X_i)the resistance andS(X_i) the stres). $G(x_i) > 0$ indicates the state of safety $G(x_i) \leq 0$ while reflects the state of failure.

The objective is to evaluate a probability of failure, Pf that of being in a situation of failure. Within the framework of the first-order approximation, the calculation of Pf is equivalent to the evaluation of an indicator called the reliability index β, the probability of failure of the system can be expressed as:

$$P_f = p_r[G(X) \leq 0] = \Phi(-\beta) \qquad \text{(IV.14)}$$

The reliability software PHIMECA Soft® can be applied to calculate the reliability index β and failure probability, where Φ(.) is the cumulative Gaussian probability function in the die environment and Pr[.] is the probability function, there are four primary factors that contribute to determining the reliability index. These factors include material rheological parameters, die temperature, fatigue damage parameters, and strain, all of which are expressed as random variables.

Table IV6: Random elements and their associated variables for fatigue and damage

Type of var.	Symbol	Description	Probability distribution model	Mean value	Coefficient of variation (%)	Source
Fatigue param.	$\mathbf{a_0}$	Constant material	Normal	0.01018	11.78	[104]
	$\boldsymbol{\beta}$	Coefficient of the damage model	Determinist	2.94	1	[116]
	$\mathbf{K_{IC}}$	fracture toughness	Normal	83.6	5.98	[104]
	C	Paris constants	Normal	$\mathbf{3.13 \times 10^{-}}$	12.65	[117]
Damage param.	Dc	Critical damage	Normal	0.4	16.03	[112,113]

The uncertainties associated with the die are linked to various states, including geometry, loading, manufacturing, and service conditions. The results obtained are presented in Tables 6 and 7, which separately list the random variables and their corresponding variables. Table.6 contains the fatigue and damage parameters, while Table.7 presents the rheological law parameters for the selected random variables during the initial extrusion phase (as per Equation IV.11).

Table IV7: Random elements and their associated variables of the rheological law.

Type of var.	Symbol	Description	Probability distribution model	Mean value	Coefficient of variation (%)	Source
Rheological law Param.	A	Material coherence	Determinist	2821.246	1	baseFPD1.3 FORGE®
	m_1	Sensitivity of material to temperature	Determinist	0.0029	1	baseFPD1.3 FORGE®
	m_2	Sensitivity of material to stress	Determinist	-0.10727	1	baseFPD1.3 FORGE®
	m_3	Sensitivity of material according to strain rate	Determinist	0.13444	1	baseFPD1.3 FORGE®
	m_4	Sensitivity of material to strain	Determinist	-0.0462	1	baseFPD1.3 FORGE®
	$\boldsymbol{\varepsilon}$	Equivalent strain resulting from the first extrusion cycle	Normal	0.04	10	baseFPD1.3 FORGE®
	T	Die temperature	Normal	485	9.27	[118]

IV.4. Variable Sensitivities

Variable sensitivities are crucial in understanding the impact of random variables on die lifetime. In **(figure IV.30**), we can observe the variable sensitivity α^2 for an accepted reliability index value of 3.7273, with the number of extruded billets increased to 1157. There are five primary elements considered in regulating die lifetime: die temperature (T), die material rheological characteristics, equivalent plastic strain after one extrusion cycle, critical damage (Dc), and fatigue damage associated variables.

Among these factors, critical damage (Dc) stands out as the most significant, contributing to 79% of all global factors influencing die life. Paris constants, accounting for less than 7%, follows in importance. Equivalent plastic strain represents 4% of the impact, with the remaining percentage attributed to all other factors. This analysis underscores the critical role of Dc in die life determination, with other factors playing a comparatively lesser role.

The reliability assessment proceeds in two main steps:

IV.4.1. Comparison of Time-Based Life Prediction Models:

Initially, the assessment centers on comparing time-based life prediction models and evaluating the parameters within the applied model. This step aims to determine how sensitive the model is to uncertainties associated with the die.

IV.4.2. Analysis of Parameter Sensitivity in the Mechanical Model:

In the second step, the assessment delves into the sensitivity of the parameters within the mechanical model, taking into account both the data of the random variables and deterministic factors **(figure IV.30**).

These steps collectively provide insights into the reliability of the die and its susceptibility to various uncertainties and factors.

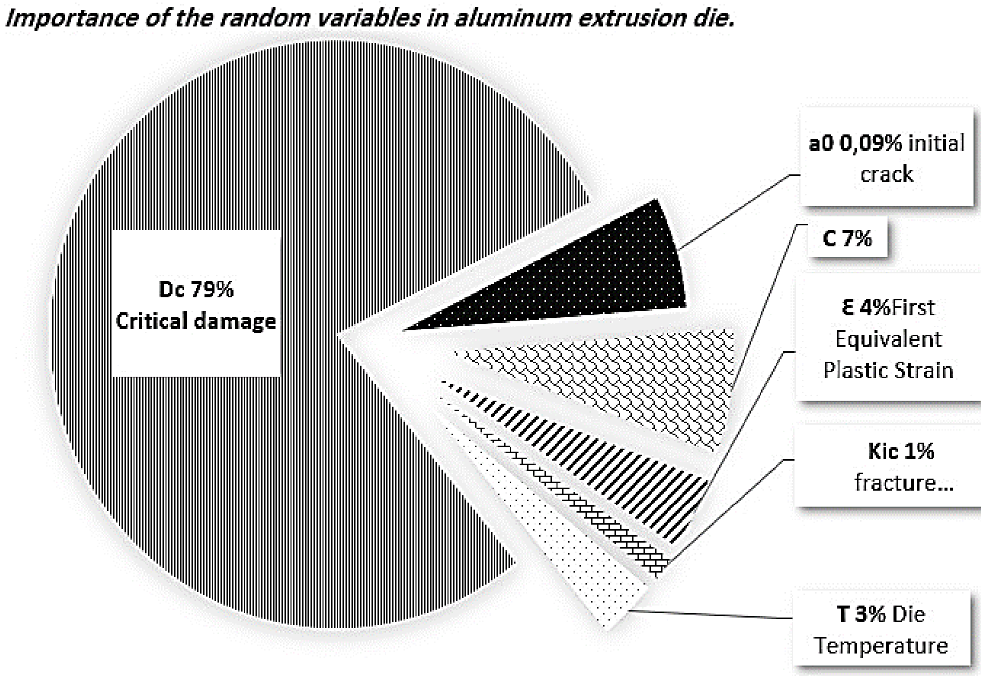


Figure IV.30: The significance of the variables in a hot aluminum extrusion die (random variables and parameters founded on Table IV.6).

(figure IV.31) illustrates the sensitivity values for die life are evaluated with respect to a range of geometrical parameters, including fracture toughness, initial crack size, maximum stress, and material-specific constants, following the previously described method. Die temperature remains a critical variable in extrusion, as it can be significantly affected during the extrusion process. To minimize uncertainties in the work described, meticulous care should be exercised when selecting material parameters and the mechanical model that governs die life. It is noteworthy that, for a reliability index (β) value of 3.72, there is a noteworthy observation regarding the lifetime number. This observation underscores the significance of managing die temperature and the careful selection of material parameters and mechanical models to ensure desired reliability levels in the extrusion process.

The impact of temperature on the die during the extrusion process is depicted in **(figure IV.32**). These figures are correlated with the value of the reliability index and the lifetime achieved under specific conditions. In this analysis, a 15% variation in fatigue parameters and a 10% variation in damage parameters were considered.

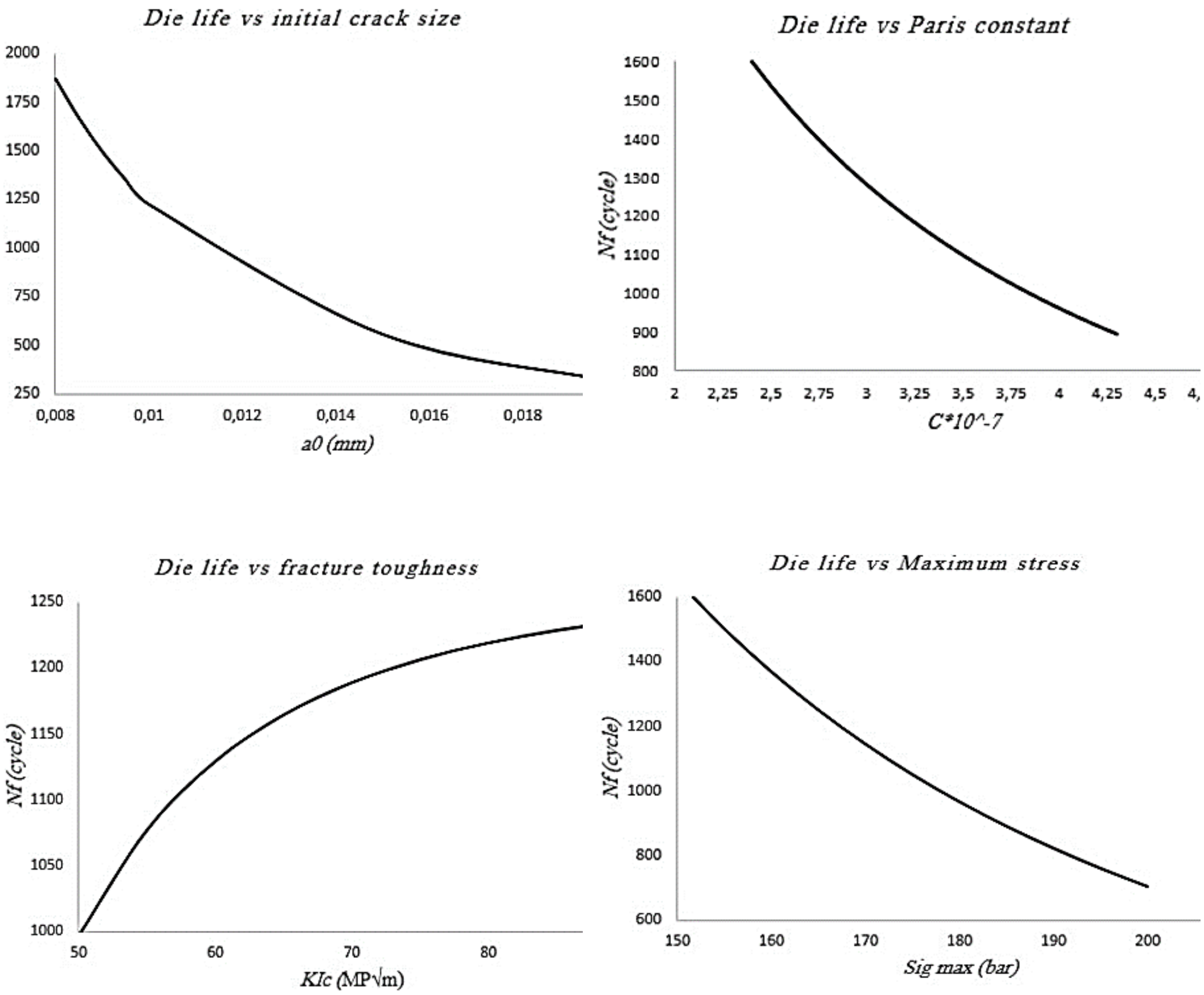


Figure IV.31: Variation of die life against geometrical parameters

Additionally, the first equivalent strain was increased from 0.06 by a coefficient variation of 10% in the die temperature, which was maintained at 511°C. These findings provide valuable insights into how changes in temperature, fatigue parameters, and damage parameters can affect the reliability index and lifetime of the die during extrusion.

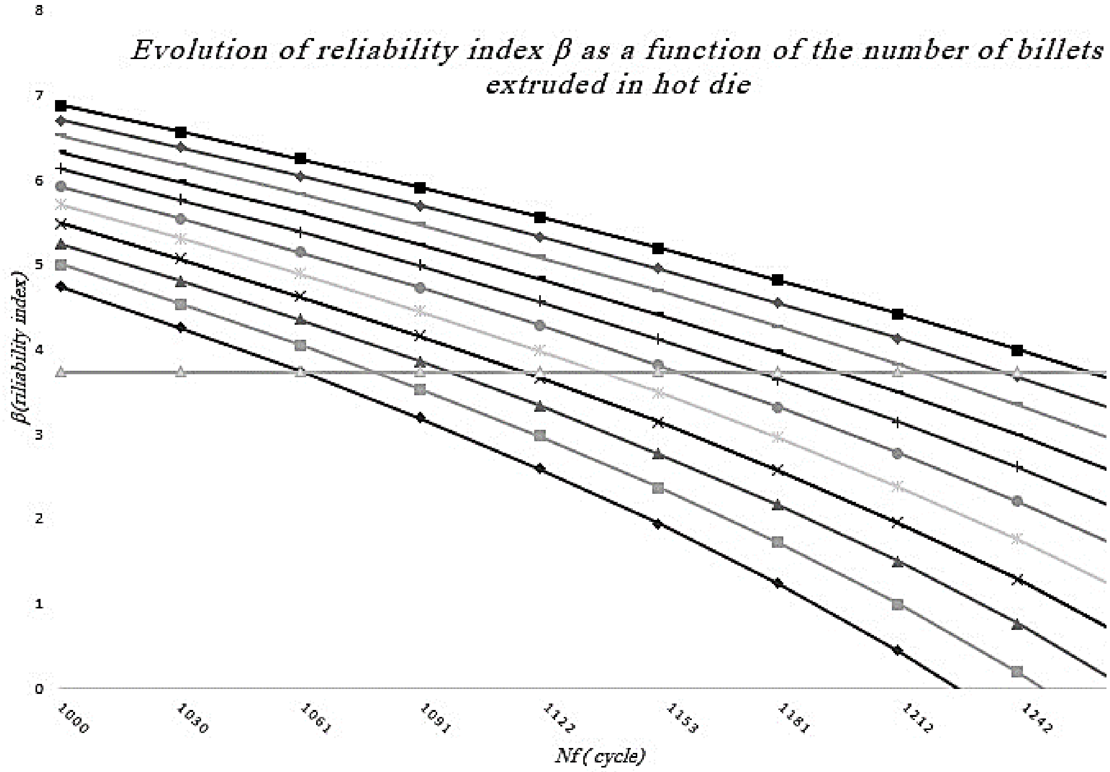


Figure IV.32: Evolution of reliability index β as a function of the number of billets extruded in hot die

The analysis presented in (**Figure IV.33)** demonstrates the evolution of the reliability index (β) at a level of 3.72. It's noteworthy that under these conditions, the cycle number of extruded billets increases to 1157 pushes when the die temperature is maintained at 511.11°C. This result represents a favorable average of the key parameters, indicating improved die performance and a longer operational lifetime for extrusion processes at this temperature.

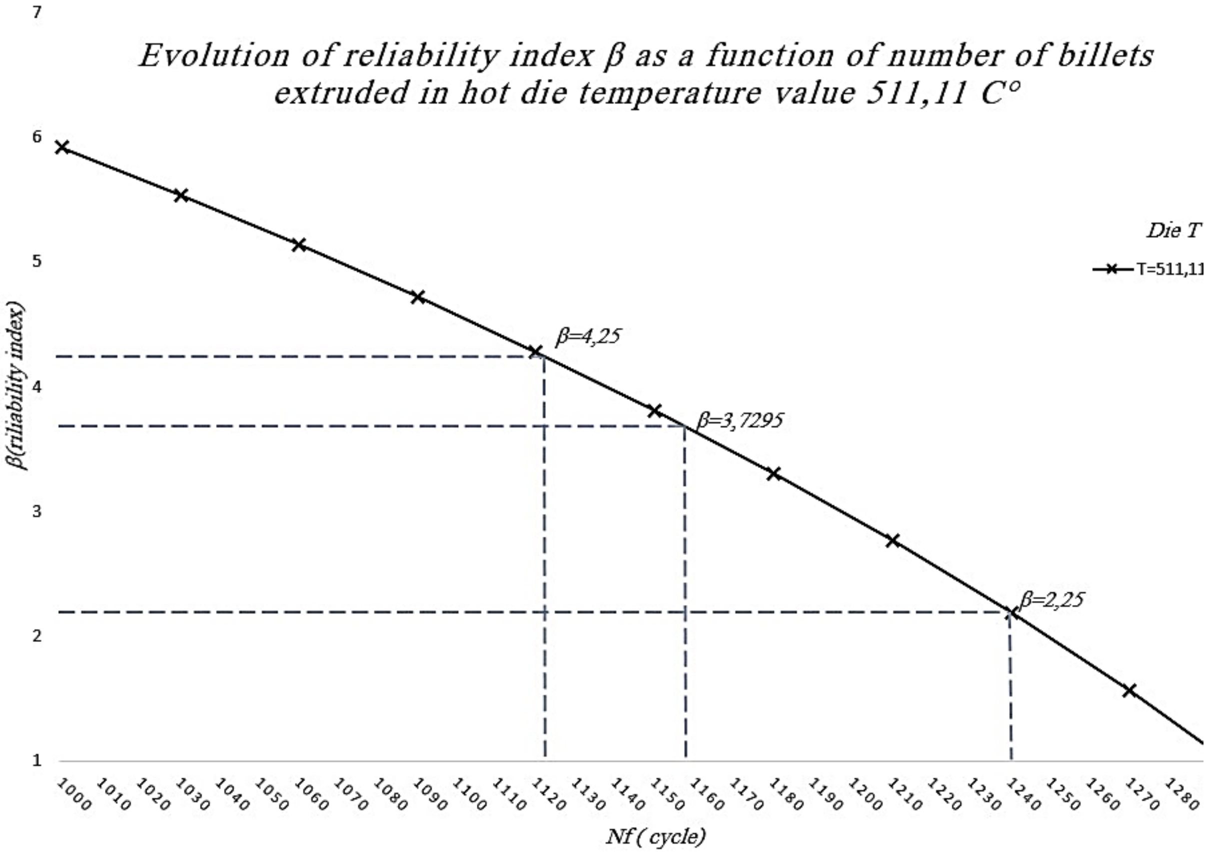


Figure IV.33: Evolution of reliability index β as a function of the number of billets extruded in hot die temperature value T=511.11°C.

IV.5. Conclusion

This chapter presents a comprehensive mechanical model to evaluate the reliability and service life of aluminum extrusions. By integrating the Hansel-Spitel rheological model with the Lemaitre-Chabusch damage model, a unified mechanistic approach to determine the reliability index β and analyze the effect of random fluctuations in process parameters is developed.

The reliability index β was calculated using the boundary state function G(Xi) = Dc − D(Xi), where Dc is the critical damage threshold and D(Xi) is the damage acting in function of material properties, fatigue parameters and matrix. Temperature and equivalent plastic stress. Probabilistic reliability assessment was performed using PHIMECASoft software.

Sensitivity analyzes revealed that the critical damage factor Dc had the most significant effect on die life, contributing 79% to the overall factors affecting reliability. Other key parameters included Paris constants for fatigue crack growth, equivalent plastic stress, and die temperature. Die temperature, in particular, has emerged as a critical variable, with careful temperature management being essential to maximize die life.

For a target reliability index of $\beta = 3.72$, the analysis predicted a favorable life for 1,157 extruded billet segments at the optimum die temperature of 511.11°C, assuming average values of the inlet parameters. This result highlights the importance of precise temperature control and precise material characterization to achieve the reliability levels required in aluminum extrusion processes.

Overall, the integrated mechanical model and reliability evaluation approach presented in this chapter provides valuable information for improving die performance, predicting failure risks, and informing maintenance strategies in the aluminum extrusion industry. Additional research could focus on improving model accuracy, accounting for additional sources of uncertainty, and exploring potential applications in other metal forming processes.

GLOBAL CONCLUSION

Global Conclusion

In this thesis, we introduce an innovative approach to evaluating the mechanical performance and predict the working life of aluminum extrusion dies exposed to thermal and cracking stresses. A comprehensive mechano-reliability framework is outlined where advanced mechanical modeling approaches are coupled with the relevant probabilistic approaches. This framework is intended to deal with the inherent variability due to material properties, loading conditions, and the manufacturing processes.

The integrated mechanical equation that comprises the Hansel-Spittel constitutive equation, the Lemaitre-Chaboche damage law, and probabilistic reliability analysis has been successfully employed to simulate die failure due to cracking under thermal loads as a result of conflicting mechanisms. In addition, the application of deterministic model and statistical analytical methods facilitate a clearer picture of the different types of failure mechanisms and the variability that accompanies them.

Sensitivity experiments under the framework were conducted to investigate the main effects on the general reliability and lifetime of the die of parameters including critical damage threshold, Paris constants for fatigue crack propagation, equivalent plastic strain, and die temperature. These realizations provide great help in die design, material selection, and operating condition optimization to increase performance and lower failure risks.

Using the probabilistic techniques offered, a mechanical engineering model has been effectively constructed with the aim of extending the lifetime of extrusion dies. Paris and rheological rules form the basis of this model. By means of a comprehensive analysis of many geometrical and material characteristics, this model has helped us to significantly improve the die's dependability. The dependability index of an extrusion die is influenced by several parameters; temperature and equivalent strain are very crucial in generating damage areas, mostly related to the propagation of fatigue cracks. It is essential to take into account the uncertain-ties of the model's parameters in order to conduct a thorough evaluation of its reliability. The comprehensive study presented in Figures 22 and 33 demonstrates that temperature, initial equivalent strain, and model parameters have a greater influence on die safety compared to variations in other components and geometric accuracy. The advanced mechanical model provides precise guidelines that establish a mathematical relationship between.

significant extrusion parameters and the number of cycles before the need for die replacement or maintenance, as evidenced by the restricted statistical data. The fatigue characteristics of the die material, the parameters of the rheological law, and other damage parameters are correlated to determine the lifetime. Through the integration of simulations and the reliability index, we anticipate the capacity to forecast operational constraints. This will provide die designers with crucial guidance on how to prolong the lifetime of extrusion dies.

The study investigated the probabilistic characteristics of failures aluminum extrusion dies used in commercial applications, focusing on the influence of the material properties. The aluminum extrusion business has experienced significant advantages from the data-driven insights acquired to enhance productivity and competitiveness. These findings suggest that meticulous parameter control and adjustment, in line with service and operating requirements, can result in the most favorable lifetime of the die. Implementing this method will minimize potential sources of failure and damage, while simultaneously enhancing the reliability and productivity of the extrusion processes. The operational and best practices perspective has covered various aspects of the extrusion process and die mechanics, such as thermal alignment, process control, extrusion metallurgy, quenching, and more.

By utilizing sophisticated computational tools such as the PHIMECASoft software, the mechano-reliability technique facilitated the computation of the reliability index β and the estimation of the chance of die failure. For average values of the input parameters at a target reliability index $\beta = 3.72$, the analysis projected an amazing die life of 1157 extruded billets with an optimal die temperature of 511.11°C. Results of this study show the capacity of the suggested framework in offering advice on maintenance strategies and lowering plant downtime while guaranteeing product quality in the aluminum extrusion sector.

Beyond the aluminum extrusion sector, the findings of this thesis reflect a flexible approach applicable for many sectors and circumstances where structural components are under thermal pressure and cracking is involved. Among other sectors including aerospace, power generation, and chemical processing, the mechano-reliability technique can be extended and changed to satisfy unique challenges.

Future directions of research could be improving the accuracy of the model by including additional sources of uncertainty, looking at advanced computing techniques for effective dependability assessment and optimization, and including real-time monitoring and data-driven approaches to support adaptive maintenance strategies.

This thesis offers a significant progress in the subject of mechano-reliability by offering a complete framework for handling the fundamental problem of die failure in aluminum extrusion processes. It also provides a foundation for next studies and progress in the larger field of structural reliability under complicated loads. The methods and discoveries described in this paper open chances for higher efficiency, lower maintenance costs, better product quality, and more economic benefits in the industrial sector and outside.

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ANNEXES

The PHIMECA Software is created by the PHIMECA organization and has been commercially available since 2002. This opportunity is available to both academics and industrialists. The interface system enhances job productivity and efficiency. Prior to conducting a dependability research, it is necessary to establish three distinct types of models:

1) A physical (or mechanical) model

The mechanical model involves formulating a mathematical law that dictates the behavior of a material or construction during its use. For the analysis of the life of the forging shaping tool, a mechanical model was created by integrating the rheological behavior of the tool, specifically the Hansel and Spittel law (equation IV.6), and a material damage law, namely the Lemaitre and Chaboche law for fatigue cracking (equation IV.3). PhimecaSoft requires users to input the mechanical model (Figure A.1).

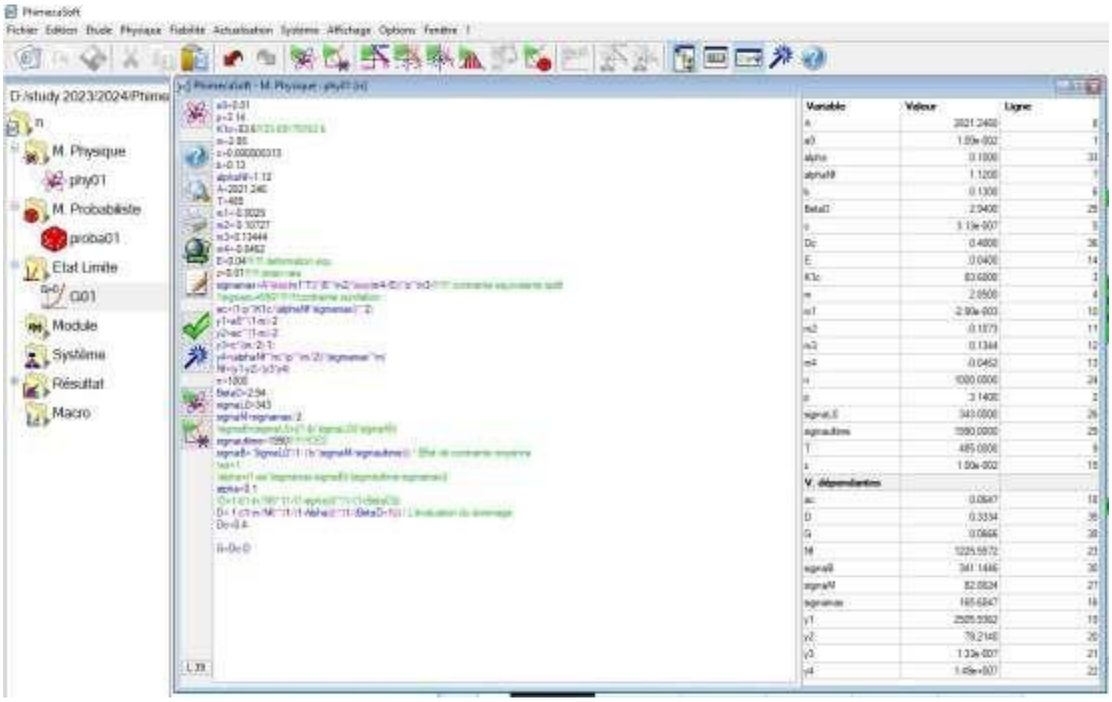
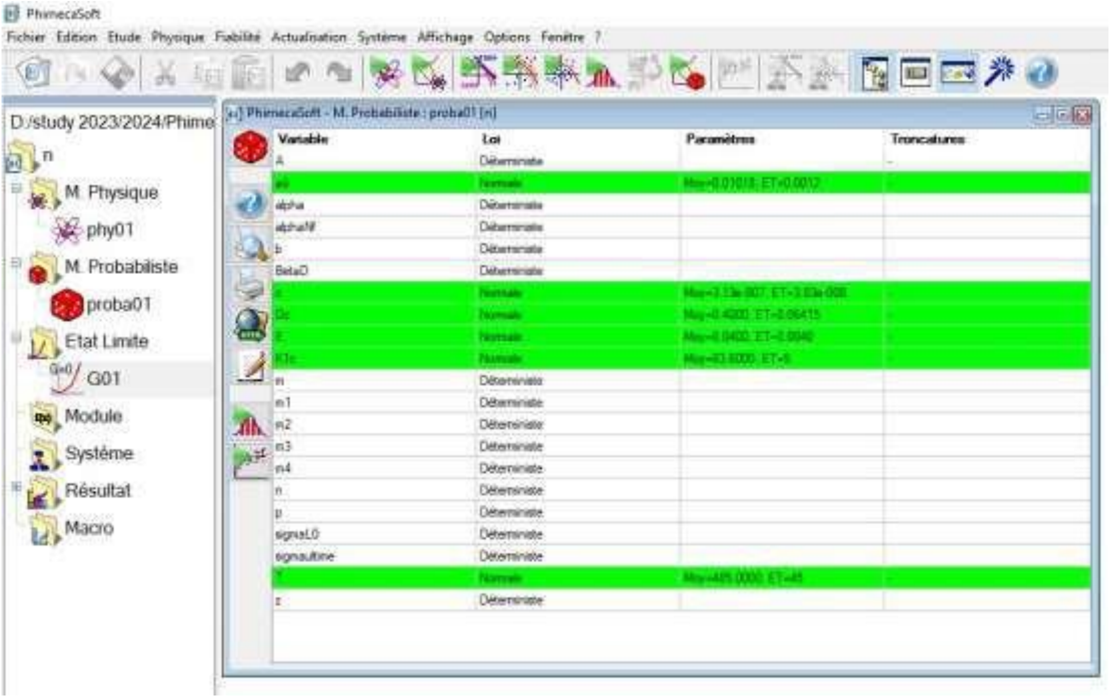


Figure A.1: mechanical model

2) A probabilistic model

The parameters of the mechanical model are represented by values that are accompanied by uncertainties. The PhimecaSoft 'Probabilistic Model' module, which provides a comprehensive selection of probabilistic laws, incorporates these uncertainties through probabilistic distribution functions. An example of inputting these probabilistic laws is illustrated in figure A.2.



Variable	Loi	Paramètres	Troncatures
A	Déterministe		-
a0	Normale	Moy=0.01018; ET=0.0012	-
alpha	Déterministe		
alphaNf	Déterministe		
b	Déterministe		
BetaD	Déterministe		
c	Normale	Moy=3.13e-007; ET=3.03e-008	-
Dc	Normale	Moy=0.4200; ET=0.06415	-
E	Normale	Moy=0.0400; ET=0.0040	-
KIc	Normale	Moy=43.6000; ET=5	-
m	Déterministe		
m1	Déterministe		
m2	Déterministe		
m3	Déterministe		
m4	Déterministe		
n	Déterministe		
p	Déterministe		
signaL0	Déterministe		
signaultime	Déterministe		
Y	Normale	Moy=485.0000; ET=45	-
z	Déterministe		

Figure A.2: probabilistic model

3) A limit state function (or performance model)

Calculation in Phimeca Soft cannot be performed without defining a limit state function. Therefore, in the case of analyzing the lifetime of the aluminum extrusion Die, the limit state is defined by (equation IV.12).

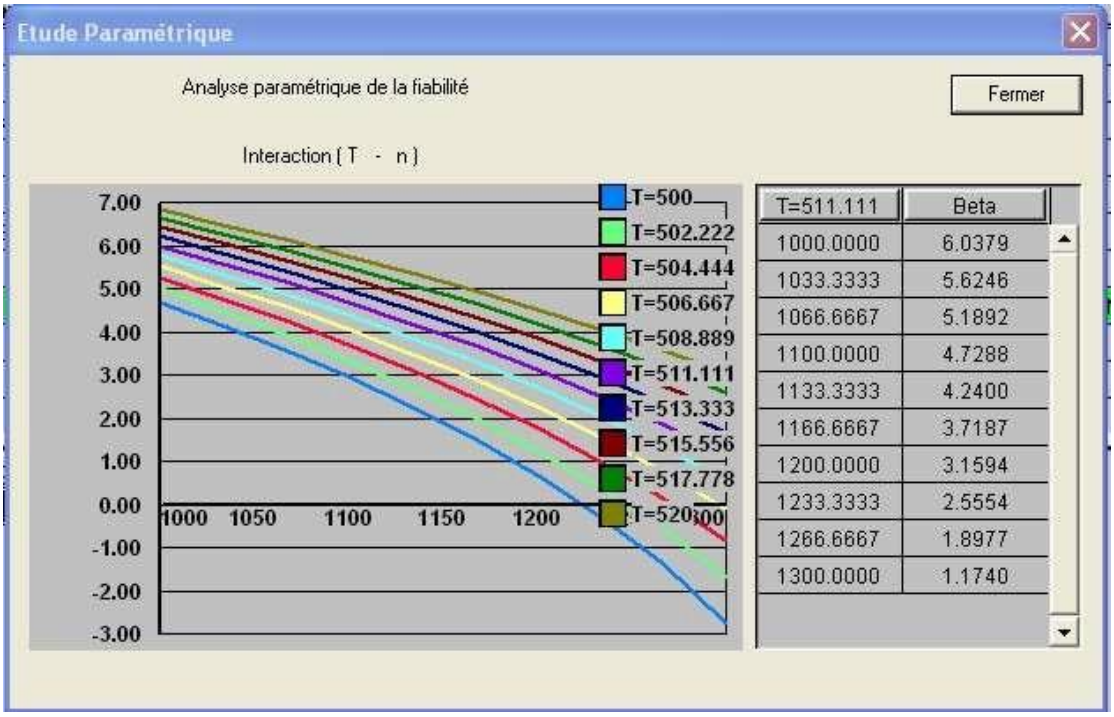
The input of this equation is simply done by indicating it in the 'Limit State' module.

4) Reliability analysis

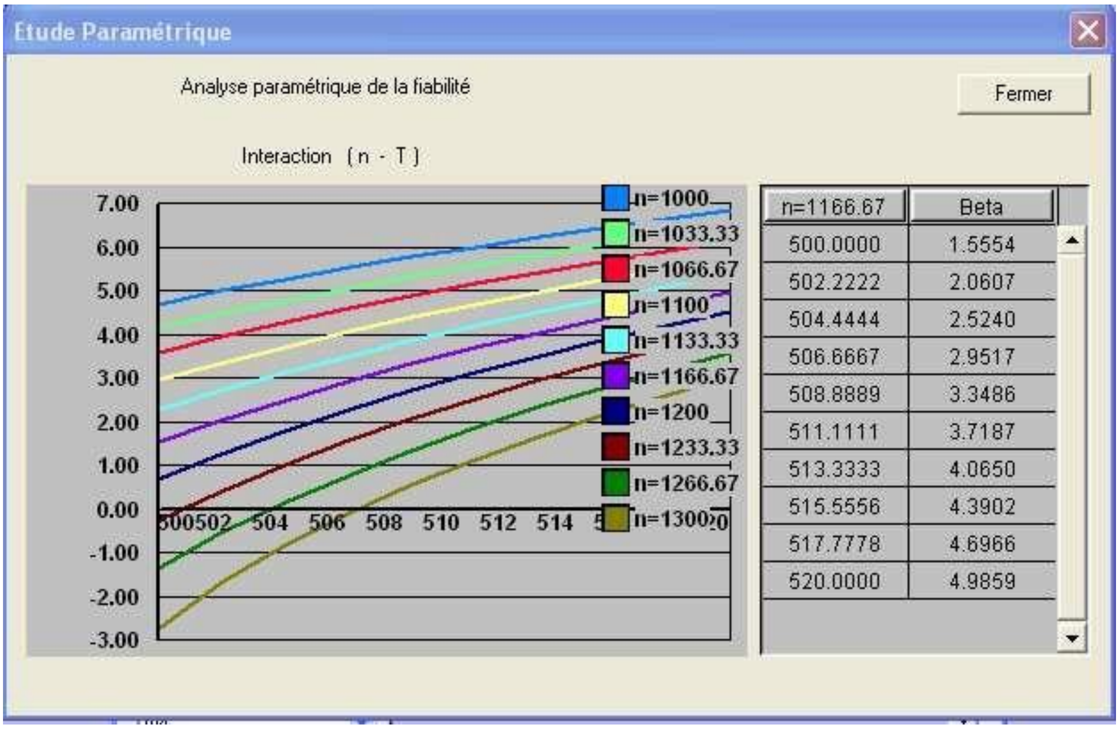
The reliability analysis can be performed in 5 ways: Approximation method (Direct), Simulations, Distribution analysis, Inverse problem, and Parametric study.

5) Reliability analysis results

The types of results that were used to carry out this work are presented below.



T=511.111	Beta
1000.0000	6.0379
1033.3333	5.6246
1066.6667	5.1892
1100.0000	4.7288
1133.3333	4.2400
1166.6667	3.7187
1200.0000	3.1594
1233.3333	2.5554
1266.6667	1.8977
1300.0000	1.1740



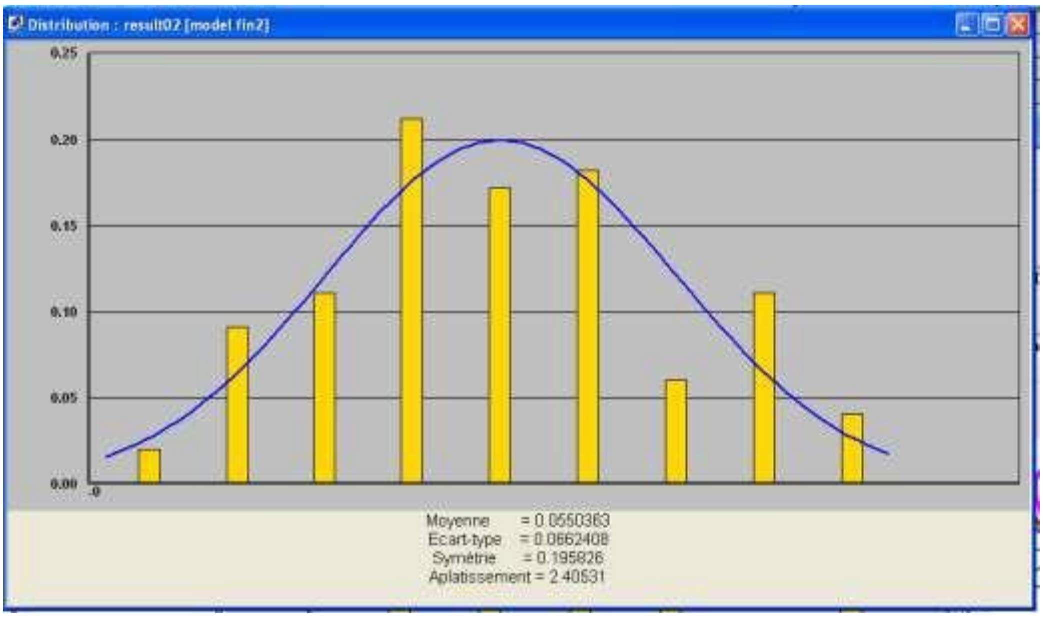
n=1166.67	Beta
500.0000	1.5554
502.2222	2.0607
504.4444	2.5240
506.6667	2.9517
508.8889	3.3486
511.1111	3.7187
513.3333	4.0650
515.5556	4.3902
517.7778	4.6966
520.0000	4.9859

Figure A.3 : Parametric study.

Deterministic parametrics before carrying out reliability studies. An example of parametric studies with variable sensitization is given in (Figure A4.3).

The parametric study makes it possible to check whether the mechanical model responds to the different parameters and their sensitivities. Thus, we can distinguish the parameters which could possibly play a determining role in the reliability analysis.

An illustration of the distribution of calculation variables obtained after a global analysis is presented in (**Figure A.4**). Using the direct method, case studies of variable sensitivity are demonstrated in (**Figures A.5**)



```
Moyenne        = 0.0550363
Ecart-type     = 0.0662408
Symétrie       = 0.195826
Aplatissement = 2.40531
```

Figure A.4 : Distribution of calculation variables

the assessment delves into the sensitivity of the parameters within the mechanical model, taking into account both the data of the random variables and deterministic factors **(figure A.5**).

These steps collectively provide insights into the reliability of the die and its susceptibility to various uncertainties and factors.

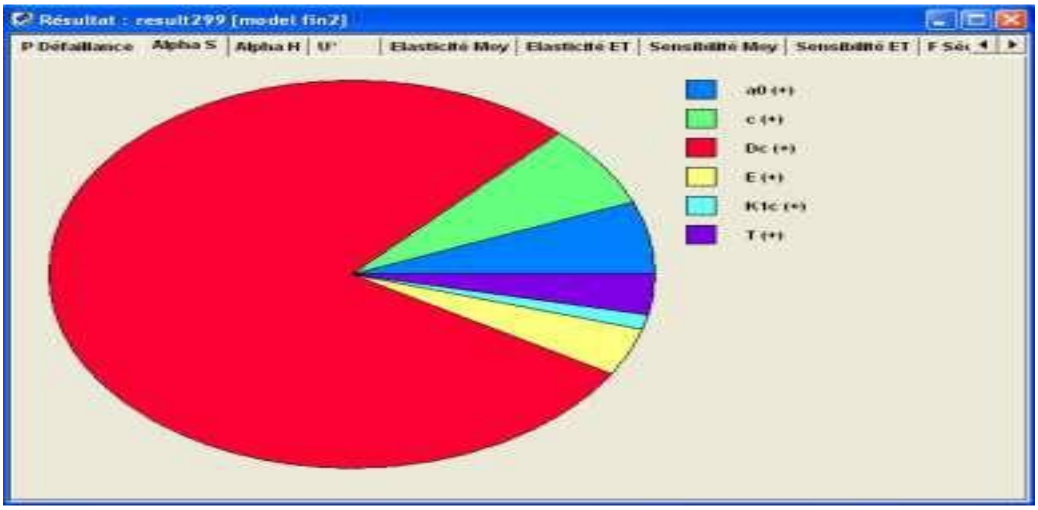


Figure A.5 : The significance of the variables in a hot aluminum extrusion die